UH - Math 4377/6308 - Dr. Heier - Fall 2011
Sample Midterm Exam I
Time: 53 min

1. (a) (10 points) Solve the equation $z(1+i) = 2+3i$ for z.

(b) (10 points) Let A, B, C be sets. Let $f : A \to B$ and $g : B \to C$ be functions. Assume that f and g are one-to-one. Prove that the composition $g \circ f$ is one-to-one.

2. (a) (10 points) Find the condition on a, b, c so that

$$(a, b, c) \in \operatorname{span}\{(1,1,0), (3,0,3), (-1,1,-2)\}.$$

(b) (20 points) Find bases for the kernel and range of $T : \mathbb{R}^5 \to \mathbb{R}^4$,
$(a_1, a_2, a_3, a_4, a_5) \mapsto (a_1 + a_3 - a_4 + a_5, -a_1 + a_2 + a_4, -a_1 + 2a_4, -a_1 + a_2 + a_3 + 2a_4 + a_5)$.

3. (a) Let $G = \{(1,2,3), (-1,1,1), (2,-2,0)\}$. Let $L = \{(1,0,1), (-1,2,1\}$.

(i) (10 points) Show that G is a basis for $\mathbb{R}^3$.

(ii) (10 points) Find a vector $v \in G$ such that $\{v\} \cup L$ is a basis for $\mathbb{R}^3$. Prove the basis property.

(b) (10 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(a_1, a_2) = (a_1 + a_2, a_1 - a_2)$. Let $\beta = \{(1,1), (-2,1)\}$ and $\gamma = \{(1,0), (1,-1)\}$. Compute $[T]_\beta^\gamma$.

4. (a) (6 points) Give an example of a linear transformation that is one-to-one, but not invertible. You are expected to justify your answer completely, so choose an example that easily allows for that.

(b) (7 points) Let V be a finite-dimensional vector space. Let $T : V \to W$ be a one-to-one linear transformation. Let V_0 be a subspace of V. Prove that $\dim V_0 = \dim T(V_0)$.

(c) (7 points) Let $\{v_1, v_2\}$ be a basis for $\mathbb{R}^2$. Is $\{v_1 + v_2, v_1 - v_2\}$ is basis for $\mathbb{R}^2$? Justify your answer carefully.