Selected Solutions HW 10

FALL 2017
Dr. Heier – Adv. Linear Algebra

#1 §4.1 Problem 3

Note: Practice a couple of these; too many students still don't know the rules for the arithmetic of imaginary numbers.
See Appendix D in the book.

#3 §4.2 Problem 4: Find the value k that satisfies the following Equation:

$$\det\begin{pmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{pmatrix} = k\det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

See theorem 4.3 (pg 212). This theorem plus the "Summarized rules" on pg 217 should be enough, let's see.

- First I will swap rows twice, leaving the determinant unchanged

$$\det(\cdot) = \det\begin{pmatrix} a_1+b_1 & a_2+b_2 & a_3+b_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ c_1+a_1 & c_2+a_2 & c_3+a_3 \end{pmatrix} \xrightarrow{R_3=R_3-R_1} \begin{pmatrix} a_1+b_1 & a_2+b_2 & a_3+b_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ c_1-b_1 & c_2-b_2 & c_3-b_3 \end{pmatrix}$$

$$\xrightarrow[R1=R1-R2]{R3=R3+R2} \begin{pmatrix} a_1-c_1 & a_2-c_2 & a_3-c_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ 2c_1 & 2c_2 & 2c_3 \end{pmatrix} \xrightarrow{R1=R1+\frac{R3}{2}} \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ 2c_1 & 2c_2 & 2c_3 \end{pmatrix}$$

* Now I will use thm 4.3 to separate the second row. →

$$\det(\cdot) = \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 2c_1 & 2c_2 & 2c_3 \end{pmatrix} + \det\begin{pmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ 2c_1 & 2c_2 & 2c_3 \end{pmatrix}$$

→ This matrix is singular since $2R2 = R3 \Rightarrow \det = 0$ →

#3 cont...

This leaves ~~us~~ us with $\det(\cdot) = \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 2c_1 & 2c_2 & 2c_3 \end{pmatrix} \underset{\text{factor out the 2}}{=} 2\det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$

$\therefore k = 2.$

* Note: You could have applied theorem 4.3 from the start, or likewise, never used it. I chose the combination as it was easiest for me to see and it showed you guys that there is another way.

#4 § 4.2 Problem 12: Calculate the determinant along the fourth row.

$$A = \begin{pmatrix} 1 & -1 & 2 & -1 \\ -3 & 4 & 1 & -1 \\ 2 & -5 & -3 & 8 \\ -2 & 6 & -4 & 1 \end{pmatrix} \begin{array}{l} \\ \\ \\ \leftarrow \text{here} \end{array}$$

$$\det(A) = 2\underbrace{\begin{vmatrix} -1 & 2 & -1 \\ 4 & 1 & -1 \\ -5 & -3 & 8 \end{vmatrix}}_{A_1} + 6\underbrace{\begin{vmatrix} 1 & 2 & -1 \\ -3 & 1 & -1 \\ 2 & -3 & 8 \end{vmatrix}}_{A_2} + 4\underbrace{\begin{vmatrix} 1 & -1 & -1 \\ -3 & 4 & -1 \\ 2 & -5 & 8 \end{vmatrix}}_{A_3} + \underbrace{\begin{vmatrix} 1 & -1 & 2 \\ -3 & 4 & 1 \\ 2 & -5 & -3 \end{vmatrix}}_{A_4}.$$

$\det(A) = 2A_1 + 6A_2 + 4A_3 + A_4$. Now must find the determinants of $\{A_i\}$'s.

~~$A_1 = -1(11) - 2(37) - (-7) = -11 - 74 + 7 = -78$~~ oops. ~ (-) got me, a lesson to take your time and not hurry!

$A_1 = -1(5) - 2(27) - (-7) = -5 - 54 + 7 = -52$

$$\left.\begin{array}{l} A_1 = -52 \\ A_2 = 42 \\ A_3 = \cancel{\ \ } -2 \\ A_4 = 14 \end{array}\right\} |A| = 2(-52) + 6(42) + 4(-2) + 14 = \boxed{154}$$

Selected Sol'n to HW 10

#8 § 4.2 Problem 30

Let the rows of A be $a_1, a_2, \dots, a_n$.

Let the rows of B be $a_n, a_{n-1}, \dots, a_1$.

Calculate $\det(B)$ in terms of $\det(A)$.

So $A = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_i \\ \vdots \\ a_{n-1} \\ a_n \end{pmatrix}$ $B = \begin{pmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_i \\ \vdots \\ a_2 \\ a_1 \end{pmatrix}$.

My proof isn't too vigorous. More of an observation that is somewhat obvious. Many of you came to this conclusion.

If n is even then we are able to split A and B down the middle, like this

$$A = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_* \\ \vdots \\ a_n \end{pmatrix} \quad B = \begin{pmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_* \\ \vdots \\ a_1 \end{pmatrix}$$

Where the top half and bottom half have the same amount of rows. (n is even) Namely that there are $n/2$ rows in each half.

And as we swap (row Δs) across this line we notice that there will always be an even # of row changes

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{n/2} \\ \vdots \\ a_{n-1} \\ a_n \end{pmatrix} \longrightarrow \begin{pmatrix} a_n \\ a_2 \\ \vdots \\ a_{n/2} \\ \vdots \\ a_{n-1} \\ a_1 \end{pmatrix} \longrightarrow \begin{pmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_{n/2} \\ \vdots \\ a_2 \\ a_1 \end{pmatrix}$$

$\therefore$ when n is even $\det(B) = \det(A)$

But when n is odd we are always left w/ one extra swap between $a_{\frac{n+1}{2}}$ and $a_{\frac{n-1}{2}}$

$\therefore$ when n is odd $\det(B) = -\det(A)$ $\quad \therefore \det(B) = (-1)^{\frac{(n-1)n}{2}} \det(A)$. $\forall n \in \mathbb{N}$.