

Math 4377/6308 (Dr. G. Heier)

Fall 2017, Univ. of Houston

HW solutions: HW2 (selected problems)

HW2, Problem 2: Section 1.2, Problem 8

$$(a+b)(x+y) = (a+b)x + (a+b)y$$

(Axiom 7)

$$= ax + bx + ay + by$$

(Axiom 8)

$$= ax + ay + bx + by \quad \text{QED}$$

(Comm.)

HW2, Problem 5: Section 1.2, Problem 13

No. Consider $2 \cdot (7, 7)$.

a) By definition,

$$2 \cdot (7, 7) = (2 \cdot 7, 7) = \underline{\underline{(14, 7)}}$$

b) On the other hand,

$$2 \cdot (7, 7) = (1 + 1) \cdot (7, 7)$$

$$\stackrel{\text{Axiom 8}}{=} 1 \cdot (7, 7) + 1 \cdot (7, 7)$$

$$\stackrel{\text{Axiom 5}}{=} (7, 7) + (7, 7)$$

$$\stackrel{\text{Def.}}{=} (7 + 7, 7 \cdot 7)$$

$$= \underline{\underline{(14, 49)}}$$

Contradiction.

(p. 2)

HW 2, Problem 7: Section 1.2, Problem 18

No. Consider $2 \cdot (7, 7)$ (again)

a) By definition,

$$2 \cdot (7, 7) = \underline{\underline{(14, 14)}}$$

b) On the other hand,

$$2 \cdot (7, 7) = (1+1)(7, 7)$$

$$= \text{Axiom 8} \quad \cancel{\times} (7, 7) + \cancel{\times} (7, 7)$$

← Def. of scalar mult. ↑

$$= (7 + 2 \cdot 7, 7 + 3 \cdot 7)$$

$$= \underline{\underline{(21, 28)}}$$

Contradiction

HW 2, Problem 8: Section 1.2, Problem 19

No. Consider $2 \cdot (7, 7)$ (again!)

a) By definition,

$$2 \cdot (7, 7) = (2 \cdot 7, \frac{7}{2}) = \underline{\underline{(14, \frac{7}{2})}}$$

b) On the other hand,

$$2 \cdot (7, 7) = (1+1)(7, 7)$$

$$\stackrel{\text{Axiom 8}}{=} 1 \cdot (7, 7) + 1 \cdot (7, 7)$$

$$\stackrel{\text{Def.}}{=} (1 \cdot 7, \frac{7}{1}) + (1 \cdot 7, \frac{7}{1})$$

$$= (7, 7) + (7, 7)$$

$$= \underline{\underline{(14, 14)}}$$

Contradiction