UH - Math 4377/6308 - Dr. Heier - Spring 2010
Sample Midterm Exam
Time: 75 min

1. (a) (5 points) Let $X = \{1, 2, 3, 4\}$ and $Y = \{3, 4\}$. Call two subsets A, B of X equivalent if $A \cup Y = B \cup Y$. Prove that this defines an equivalence relation on the set of subsets of X.

(b) (10 points) Let $z = 1+4i$ and $w = -4-3i$. Find $|w|$. Write zw and $\frac{z}{w}$ in the form $a+bi$.

2. (a) (10 points) Determine if the following subset of $\mathbb{R}^3$ is a subspace. Justify your answer carefully:

$$\{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 - 2a_2 + a_3^2 = 0\}.$$

(b) (10 points) Let W_1, W_2 be two subspaces of a vector space V. Prove that the intersection $W_1 \cap W_2$ is also a subspace of V.

3. (a) (10 points) Find the condition on a, b, c so that

$$(a, b, c) \in \operatorname{span}\{(1, 1, 2), (3, 0, 3), (-1, 1, 0)\}.$$

(b) (10 points) Find a basis for the following subspace W of $\mathbb{R}^5$:

$$W = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 : a_1 + a_2 + a_3 + a_4 + a_5 = 0, a_2 = 2a_3 = -a_5\}.$$

4. (15 points) Find nullity and rank of

$$T : \mathbb{R}^5 \to \mathbb{R}^3, (a_1, a_2, a_3, a_4, a_5) \mapsto (a_1 + a_5, -a_1 + a_2 + a_3, 3a_1 - a_2 - a_3 + 2a_5).$$

5. (15 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(a_1, a_2) = (a_1 + 3a_2, -a_1 - a_2)$. Let $\beta = \{(1, 2), (-1, 1)\}$ and $\gamma = \{(2, 1), (2, 0)\}$. Compute $[T]_\beta^\gamma$.

6. (15 points) Let $T : V \to W$ be a linear transformation. Suppose $\beta = \{v_1, \ldots, v_n\}$ is a basis for V and T is one-to-one and onto. Prove that $T(\beta) = \{T(v_1), \ldots, T(v_n)\}$ is a basis for W.