UH - Math 4378/6309 - Dr. Heier - Spring 2011
HW 8
Due 03/23, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Give an example of a linear operator $T : V \to V$ with V a **real** inner product vector space such that T is normal, but not diagonalizable. (It is ok to give T as a matrix.)

2. (1 point) Take your example from Problem 1 and explicitly find an ONB of eigenvectors of T over the complex numbers. (Note that the existence of the ONB is guaranteed by Theorem 6.16.)

3. (1 point) Consider the linear operator on $\mathbb{R}^3$ given in standard coordinates by the matrix

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{pmatrix}.$$

Find an ONB of eigenvectors for the standard inner product on $\mathbb{R}^3$. Verify explicitly that all vectors you list are orthogonal to each other.

4. (1 point) Consider the linear operator on $\mathbb{R}^3$ given in standard coordinates by the matrix

$$A = \begin{pmatrix} 0 & -2 & -2 \\ -2 & 0 & -2 \\ -2 & -2 & 0 \end{pmatrix}.$$

Find an ONB of eigenvectors for the standard inner product on $\mathbb{R}^3$. Verify explicitly that all vectors you list are orthogonal to each other.

5. (2 points) Consider the linear operator on $\mathbb{C}^3$ given in standard coordinates by the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}.$$

Find a basis of eigenvectors over the complex numbers. Is there an ONB of eigenvectors for the standard inner product on $\mathbb{C}^3$? Why?

6. (2 points) Section 6.4, Problem 10

7. (2 points) Section 6.4, Problem 11

8. (1 bonus point) Section 6.4, Problem 16