UH - Math 4378/6309 - Dr. Heier - Spring 2011
Sample Midterm Exam
Time: 53 min

1. (a) (15 points) Find the characteristic polynomial of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

(Hint: You may use any theorem from class, but you must explain your reasoning carefully.)

(b) (10 points) State the Cayley-Hamilton Theorem and verify explicitly that the linear operator on $\mathbb{R}^4$ given by the matrix A satisfies it.

2. (25 points) Let T be a linear operator on a finite-dimensional vector space V, and let W be a T-invariant subspace. Let v_1, v_2 be eigenvectors of T corresponding to distinct eigenvalues. Assume also that $v_1 + v_2 \in W$. Prove that $v_1 \in W$ and $v_2 \in W$.

3. (a) (10 points) Consider $\mathbb{C}^3$ with the standard inner product. Let $v = (1, 0, i)$. Find an ONB for $\{v\}^\perp$.

(b) (10 points) Let V be an inner product space, and let T be a normal operator on V. Prove that, for all $x \in V$,

$$\|T(x)\| = \|T^*(x)\|.$$

(c) (5 points) Under the assumptions in part (b), is it true that for all $x \in V$, $\|T(x)\| = \|x\|$? Justify your answer carefully.

4. (a) (10 points) Let

$$A = \begin{pmatrix} 0 & 1+i \\ 1+i & 0 \end{pmatrix}.$$

Is A self-adjoint? Is A normal? Prove your answer.

(b) (15 points) Find an ONB of eigenvectors of A for $\mathbb{C}^2$.