UH - Math 6303 - Dr. Heier - Spring 2016
HW 4
Due Monday, 04/18, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (3 points) Let $f(x) \in \mathbb{Q}[x]$ be an irreducible polynomial of degree p, where p is a prime. Assume that $f(x)$ has precisely two nonreal roots in the complex numbers. Prove that the Galois group of the splitting field of $f(x)$ is the full symmetric group S_p.

2. (3 points) Let $f(x) \in \mathbb{Q}[x]$ be the polynomial $x^9 - 1$. Determine the Galois group of the splitting field of $f(x)$. Hint: You may assume without proof that the polynomial $x^6 + x^3 + 1$ is irreducible over $\mathbb{Q}$.

3. (2 points) Let K be the splitting field over F of a separable polynomial. Prove that if $\mathrm{Gal}(K/F)$ is cyclic, then for each divisor d of $[K : F]$ there is exactly one field E with $F \subset E \subset K$ auch that $[E : F] = d$. (Hint: Use the Fundamental Theorem of Galois Theory.)

4. (2 points) Suppose K/F is a Galois extension of degree p^n for some prime p and positive integer n. Prove that there are Galois extensions of F contained in K of degrees p and p^{n-1}. (Hint: Use the Fundamental Theorem of Galois Theory.)