

UH - Math 3330-01 - Dr. Heier - Spring 2017

HW 3

Due Friday, 02/10, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (2 points) Let  $G$  be a nonempty set and let  $*$  be an associative binary operation on  $G$ . Assume that for any elements  $a, b \in G$ , we can find  $x \in G$  such that  $a * x = b$ , and we can find  $y \in G$  such that  $y * a = b$ . Prove that  $(G, *)$  is a group.
2. (2 points) Let  $G$  be a group. Let  $a, b \in G$ . Prove that  $o(ab) = o(ba)$ .
3. (2 points) Prove that if  $G$  is a finite group, then every element of  $G$  is of finite order.
4. (2 points) Determine  $(561, 84)$ . Find integers  $m, n$  such that  $561m + 84n = (561, 84)$ .
5. (2 points) Let  $G$  be a group. Let  $x, y \in G$ . Assume that  $x \neq e$ ,  $o(y) = 2$ , and  $xyx^{-1} = x^2$ . Determine  $o(x)$ .