UH - Math 3330-01 - Dr. Heier - Spring 2017
HW 5
Due Friday, 02/24, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Let G be a group.

(a) (1 point) Give a complete list of all the subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_4$. You do not need to give a proof—just write down the list.

(b) (1 point) Give a complete list of all the subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. You do not need to give a proof—just write down the list.

2. Let G, H be finite groups such that $G \times H$ is a cyclic group.

(a) (1 point) Prove that both G and H are cyclic.

(b) (1 point) Prove that every subgroup of $G \times H$ is of the form $A \times B$ for a subgroup A of G and a subgroup B of H.

3. Let $f : A \to B$ be a function between non-empty sets A, B.

(a) (1 point) Prove that for all subsets $S, T \subseteq A$, $f(S \cup T) = f(S) \cup f(T)$.

(b) (0.5 points) Prove that for all subsets S, T of A, $f(S \cap T) \subseteq f(S) \cap f(T)$.

(c) (0.5 points) Give an example where the containment relation in item (b) is strict.

4. Let $f : A \to B$ be a function between non-empty sets A, B.

(a) (1 point) Prove that f is injective if and only if there exists a function $g : B \to A$ such that $g \circ f = \mathrm{id}_A$, where $\mathrm{id}_A : A \to A, a \mapsto a$ is the identity function.

(b) (1 point) Prove that f is surjective if and only if there exists a function $g : B \to A$ such that $f \circ g = \mathrm{id}_B$.

5. Let $f : A \to B$ and $g : B \to C$ be functions.

(a) (1 point) Assume that $g \circ f$ is injective. Does this imply that both f and g are injective? Prove your answer.

(b) (1 point) Assume that $g \circ f$ is surjective. Does this imply that both f and g are surjective? Prove your answer.