UH Math 3330-01 Dr.Heier-Spring 2017
HW1 Answer Key

Yan He

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Problem 1

(a)

Proof. We prove (a) by proving $T \cup (S \setminus T) \subset S \cup T$. and $S \cup T \subset T \cup (S \setminus T)$. For arbitrary $x \in T \cup (S \setminus T)$, $x \in T$ or $x \in S \setminus T$. If $x \in T$, then $x \in T \cup S$; If $x \notin T$, then $x \in S \setminus T$, then $x \in S$. So we conclude that $x \in S \cup T$. So $T \cup (S \setminus T) \subset S \cup T$.

If $x \in S \cup T$, then $x \in S$ or $x \in T$. If $x \notin T$, then $x \in S \setminus T$ follows from the definition of $S \setminus T$. So $x \in T \cup (S \setminus T)$. Hence $S \cup T \subset T \cup (S \setminus T)$. □

(b)Similar.

Problem 2

(a)

Proof. First we prove LHS⊂RHS. For every $x \in A \cap (B \cup C)$, $x \in A$ and $x \in B \cup C$. This implies $x \in B$ or $x \in C$. If $x \in B$, then $x \in A \cap B$. If $x \in C$, then $x \in A \cap C$. So is in $A \cap B$ or $A \cap C$, which means $x \in (A \cap B) \cup (A \cap C)$.

Then we prove the other direction RHS⊂LHS, if $x \in (A \cap B) \cup (A \cap C)$, then $x \in A \cap B$ or $x \in A \cap C$. If $x \in A \cap B$, then $x \in A$ and $x \in B$. If $x \in A \cap C$ then $x \in A$ and $x \in C$. Then $x \in A$ and $x \in B$ or $x \in C$. So $x \in A$ and $x \in (B \cup A)$. So $x \in A \cap (B \cup C)$. □

(b) Omitted.

Problem 3

Can use induction.

Problem 4

Proof. We use induction. When $n = 4$, clearly $2^4 = 16 < 4! = 24$.

Assume the statement is true for an integer $n \geq 4$, i.e. $2^n < n!$. Then

$$\frac{n+1}{2} \geq \frac{4+1}{2} > 1.$$

So

$$n!(\frac{n+1}{2}) > n! > 2^n$$

Then

$$n!(n+1) = (n+1)! > 2 \cdot 2^n = 2^{n+1}.$$

So we the statement is true for $n+1$. From the principle of mathematical induction we know the statement is true for all integers $n \geq 4$. $\square$

Problem 5

A binary operation on a set S can be viewed as a map from $S \times S$ to S. A commutative binary operation on a set S can be viewed as a map from $S \times S/\sim \to S$, where $\sim$ is a equivalence relation defined by $(a, b) \sim (c, d)$ if and only if $((a = d$ and $b = c)$ or $(a = c$ and $b = d))$. By counting the number of functions from finite sets A to B are $(\#B)^{\#A}$. $\#S = n$, $\#(S \times S) = n^2$ so the number of binary operations on S is n^{n^2}. $\#(S \times S/\sim) = C_{2,n} + n = \frac{n(n+1)}{2}$, where $C_{2,n}$ means the number of choices to pick 2 elements from n elements. So the number of commutative binary operations are $n^{\frac{n(n+1)}{2}}$.