UH Math 3330-01 Dr.Heier-Spring 2017
HW2 Answer Key

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Problem1.

(a) is a group.

(b) is not a group because $+$ is not a (closed) binary operation.

Remarks:

- You cannot prove a property by taking specific examples, such as $2+4=6$. However, you can do so if you want to prove that a property does not hold. This is called providing a "counterexample."

Problem2.

(a)

Proof. For every $a, b, c \in \mathbb{Z}_n$, $(a \odot b) \odot c = \overline{a \cdot b} \odot c = \overline{(a \cdot b) \cdot c} = \overline{a \cdot (b \cdot c)} = a \odot \overline{(b \cdot c)} = a \odot (b \odot c)$. □

(b) $\odot$ is not a binary operation on $\mathbb{Z}_4 \setminus \{0\}$ because $2 \odot 2 = 0 \notin \mathbb{Z}_4 \setminus \{0\}$.

(c)$\mathbb{Z}_5 \setminus \{0\}$ is a group. (Generally, $\{\mathbb{Z}_p \setminus \{0\}, \odot\}$ is a group if and only if p is prime. This will be discussed later in this course.)

Problem3.

(a) omitted.

(b) To solve $7 \odot x = 5$ for $x \in \mathbb{Z}_{13}$, one simply has to test all $x = 0, 1, 2, 3, 4, \ldots, 12$. Doing so shows that $x = 10$ is the unique solution.

Problem4.

Proof. For every $x, y \in G$, $x * y = e * (x * y) * e = (y * y) * x * y * (x * x) = y * (x * y) * (x * y) * x = y * e * e * x = y * x$. □

Problem5.

Remarks

- To prove "if and only if" you need to prove both two directions. Though in this problem the operations are invertible so essentially don't have to, a complete proof should mention this.

Proof. If G is commutative then the statement holds obviously. If for every $x, y \in G$, $(x * y)^2 = x^2 * y^2$, then $(x * y)^2 = x * y * x * y = x * (y * x) * y = x^2 * y^2 = x * x * y * y$. So

$$x * x * y * y = x * y * x * y.$$

Multiply by x^{-1} on the left and by y^{-1} on the right to the equation we get

$$x * y = y * x$$

.

$\square$