UH Math 3330-01 Dr.Heier-Spring 2017
HW3 Answer Key

Yan He

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Problem1.

Proof. Let $a = b$, then we have $x \in G$ s.t. $a * x = a$. Here the choice of x depends on a, we want to prove that x is actually a right identity, which does not depend on a. For every $b \in G$, we can find $y \in G$ s.t. $y * a = b$. Then $b * x = (y * a) * x = y * (a * x) = y * a = b$. So we have proved $b * x = b$ for arbitary $b \in G$. Then x is right identity. Denote the right identity of G by e. By solving $b * y = e$ we can find the right inverse b^{-1} for every $b \in G$. From the theorem in class we know G is "half" a group with right identity and is right invertible thus G is a group. □

Problem2.

Proof. Assume $ord(ab) = n, ord(ba) = m$.

Case 1: $n < \infty$ then $b(ab)^n = (ba)^n b$, hence $(ba)^n = e$. Then $m|n$. Same argument shows $n|m$. Thus $n = m$.

Case 2: $n = \infty$ If $m < \infty$ then from case 1 we know $n = m$, contradiction. □

Problem3.

Proof. WLOG assome G is non-trivial. For every $g \neq e \in G$, define a map $f : \mathbb{N} \to G$, $n \mapsto g^n$. G is finite so G is bijective to $\{1, 2, ...n\}$ for some $n \in \mathbb{N}$ by definition. Then $\mathbb{N}$ is bijective to a subset of $\{1, 2, ..., n\}$ so can conclude that $\mathbb{N}$ is finite, which is a contradiction. So f is not injective. There must be $m, n \in \mathbb{N}$ s.t. $f(m) = f(n)$, i.e. $g^m = g^n$. So $ord(g)|(|m - n|)$. □

Remark: I assigned 1 point for proving $g^m = g^n$. Which seems obvious.

Problem4.

Proof. omitted. □

Problem5.

Proof. First observe that $y^2 = 1$, $y = y^{-1}$, $x^2 = yxy$ then $x^4 = x^2x^2 = yxyyxy = yx^2y = yyx = x$ thus $ord(x) = 3$.

□