UH Math 3330-01 Dr.Heier-Spring 2017
HW6 Answer Key

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Problem1. (a)$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 4 & 2 & 1 & 7 & 3 & 5 \end{pmatrix}$ (b)$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 6 & 5 & 7 & 1 & 3 & 2 \end{pmatrix}$

Problem2.

(a)$(1,4,6,8,10,2,3,9)(5,7) = (1,9)(1,3)(1,2)(1,10)(1,8)(1,6)(1,4)(5,7)$;even

(b)$(1,7,8,9,2,3)(4,6,5) = (1,3)(1,2)(1,9)(1,8)(1,7)(4,5)(4,6)$;odd

Problem3. (a) $(123456789) = (19)(18)(17)(16)\cdot(15)(14)(13)(12) \in S_9$ is order-9 element, can choose $x = (16789), y = (12345)$.

(b)In general, the largest order in S_n is a hard problem, see [1]. Well, since S_9 is relatively small, the largest order is 20 by for example (12345)(6789) by checking all possibilities.

Problem4.

Let

$$x = ...(-3,-2)(-1,0)(1,2)(3,4)(5,6)...$$

$$y = ...(-4,-3)(-2,-1)(0,1)(2,3)(4,5),...$$

Then x and y are obviously order 2. While $xy = (... -1, -2, 0, 2, 4, 6, 8, ...)$ is infinite order.

Problem5.

(a)

Proof. For every $x \in G$ we have $x = exe^{-1}$ so it is reflexive.
If $a \sim b$ then $\exists x \in G$ s.t. $a = xbx^{-1}$ by definition, which implies $b = x^{-1}ax = x^{-1}a(x^{-1})^{-1}$, so $b \sim a$ by definition, $\sim$ is symmetric. If $a \sim b, b \sim c$ then $\exists x \in G$ s.t $b = xax^{-1}$ and $c = yby^{-1} = y(xax^{-1})y^{-1} = (yx)a(yx)^{-1}$. which implies $a \sim c$. □

(b)

Proof. For integer x let $[x]$ denote the equivalent class of x in $\mathbb{Z}_7$. Then $x \sim y$ if and only if $[3x - 10y] = [0] \iff [3x] - [3y] - [7y] = [0] \iff [x] = [y]$. So $\sim$ is precisely the congruence relation, which is an equivalence relation. □

References

[1] Miller, William. "The maximum order of an element of a finite symmetric group." Amer. Math. Monthly 94, no. 6 (1987): 497-506.