UH Math 3330-01 Dr.Heier-Spring 2017
HW7 Answer Key

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March 17, 2017

Problem1. (a)Not an equivalence relation because transitivity fails.
(b)Is an equivalence relation, obviously.

Problem2.
(a)H,$H+(0,1)=\{(0,1),(1,1),(2,1)\}$
(b)H,
$H+(0,3)=\{(0,3),(0,1)\}$,
$H+(1,2)=\{(1,2),(1,0)\}$,
$H+(1,3)=\{(1,3),(3,1)\}$
$H+(2,2)=\{(2,2),(2,0)\}$,
$H+(2,3)=\{(2,3),(2,5)\}$,
$H+(3,2)=\{(3,2),(3,0)\}$
$H+(3,3)=\{(3,3),(3,1)\}$

Problem3.
Follows from the Lagrange's theorem, and the fact that every group of prime order must be cyclic.

Problem4.

Proof. (i)If G has one element $g \in G$ of order p^2, then G must be cyclic because $|<g>|=|G|$ and $<g>$ is a subgroup of $G \implies <g>=G$. Then g^p is the element of order p, thus we can find a subgroup of order p.

(ii)If G has no element of order p^2, then every element of G must have order $1 or p$ by Lagrange theorem. But G must have at least one element of order g which generates a cyclic subgroup of G of order p, otherwise G would be $\{e\}$, which contradicts $|G|=p^2$. □

Problem5.

Proof. $(x_1 \cdot ... \cdot x_{r-1})^2 = x_1 x_1^{-1} \cdot x_2 x_2^{-1} \cdot ... \cdot x_{r-1} x_{r-1}^{-1} = e$. Then the order of $(x_1 \cdot ... \cdot x_{r-1})$ divides 2. But $|G|$ is odd, so the order can only be one, which means $(x_1 \cdot ... \cdot x_{r-1}) = e$. □