UH Math 3330-01 Dr.Heier-Spring 2017
HW Answer Key

Yan He

Problem1. The isomorphism is given by $\phi : G \times H \to H \times G, (g, h) \mapsto (h, g)$.
Problem2. $H = \{(0,0),(0,2)\}$, $K = \{(0,0),(1,0)\}$. Then $G/H = \mathbb{Z}_2 \times \mathbb{Z}_2$ and $G/K = \mathbb{Z}_4$. Note that they aren't isomorphic because there is not an element of order 4 in $\mathbb{Z}_2$ but there is one in $\mathbb{Z}_4$.
Problem3. $Aut(G)$ is a subset of S_G obviously. Then to show $Aut(G)$ is a subgroup we need to show $Aut(G)$ is closed under composition and inverse. For $f, g \in Aut(G)$, $f \circ g$ is clearly a bijection, so we just need to show $f \circ g$ is a homomorphism. For every $r, s \in G$,

$$f \circ g(rs) = f(g(rs)) = f(g(r)g(s) = f(g(r))f(g(s)) = f \circ g(r) f \circ g(s)$$

Thus $f \circ g$ is a homomorphism. Note that $id_G \in Aut(G)$. $Aut(G)$ is closed under inverse by Thm12.1(iii).
Problem4. $\mathbb{Z}$ is a cyclic group, for every $f \in Aut(G)$ we must have $f(1) = \pm 1$ Note that $Aut(\mathbb{Z})$ has only two elements, and groups with two elements has only one structure.
Problem5. This is not true. Assume $\phi \in Aut(G)$, then ϕ is homomorphism. For $r, s \in G$, if $r \in H, s \notin H$, then $rs \notin H$. $\phi(rs) = rs = \phi(r)\phi(s) = \psi(r)s$, thus $\psi(r) = r$. This can only happen when $\phi = id_H$. Of course not every automorphism is identity map.