UH - Math 3330 - Dr. Heier - Spring 2019
HW 11
Due MONDAY, 04/29, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (1 point) Prove that $\mathbb{Z}_n$ is a field if and only if n is a prime number.

2. (1 point) Let R be a ring with unity. Assume that for all x and y in R we have $(xy)^2 = x^2y^2$. Prove that R is commutative.

3. Let $S = \{q \in \mathbb{Q} : q = \frac{a}{b},\ a, b \in \mathbb{Z} \text{ and } b \text{ odd}\}$.

(a) (0.5 points) Prove that S is a subring of $\mathbb{Q}$.

(b) (0.5 points) Prove that S has a unique maximal ideal.

4. Let R be a commutative ring with unity $1 \neq 0$.

(a) (0.5 points) Prove that R is an integral domain if and only if $\{0\}$ is a prime ideal in R.

(b) (0.5 points) Prove that R is a field if and only if $\{0\}$ is a maximal ideal in R.

5. (2 points) Let I be an ideal in the commutative ring R. Define

$$\text{rad}(I) = \{r \in R \mid \exists n \in \mathbb{N} : r^n \in I\}.$$

Prove that $\text{rad}(I)$ is an ideal with $I \subset \text{rad}(I)$.

6. (2 points) Let R be a commutative ring and $I \subset R$ a prime ideal. Prove that $\text{rad}(I) = I$.

7. Which of the following is a ring homomorphism? Prove your answer.

(a) (1 point) $\varphi : \mathbb{R} \to \mathbb{R}, \varphi(x) = |x|$,

(b) (1 point) $\varphi : \mathbb{C} \to \mathbb{C}, \varphi(a + ib) = a - ib$.