UH - Math 3330 - Dr. Heier - Spring 2019
HW 3
Due Wednesday, 02/06, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. Let $G = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\} \subset \mathbb{R}$.

(a) (1 point) Prove that G is a group under addition.

(b) (1 point) Prove that the non-zero elements of G are a group with multiplication.

2. (2 points) Let G be a nonempty set and let $*$ be an associative binary operation on G. Assume that for any elements $a, b \in G$, we can find $x \in G$ such that $a * x = b$, and we can find $y \in G$ such that $y * a = b$. Prove that $(G, *)$ is a group. Carefully write the proof in your own words.

3. (2 points) Let G be a group. Let $x \in G$. Prove that $o(x) = o(x^{-1})$.

4. (2 points) Let G be a group and let $x \in G$ be of finite order n. Prove that if n is odd, then there exists a k such that $x = x^{2k}$ for some $k \geq 1$.

5. (2 points) Determine $(561, 84)$. Find integers m, n such that $561m + 84n = (561, 84)$.

6. (0 points) Let G be a group. Let $x, y \in G$. Assume that $x \neq e$, $o(y) = 2$, and $yxy^{-1} = x^2$. Determine $o(x)$.