UH - Math 3330 - Dr. Heier - Spring 2019
HW 4
Due Wednesday, 02/13, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (2 points) Find the order of $30 \in \mathbb{Z}_{54}$. Write down $\langle 30 \rangle$. You don't need to give any proofs in your solution.

2. Let G be a group.

(a) (1 point) Prove that if $G = \langle x \rangle$, then $G = \langle x^{-1} \rangle$.

(b) (1 point) Prove that if $G = \langle x \rangle$ and G is infinite, then x and x^{-1} are the only generators of G.

3. Let H, K be subgroups of a group G.

(a) (1 point) Prove that $H \cap K$ is a subgroup of G.

(b) (1 point) Prove that $H \cup K$ is a subgroup of G if and only if ($H \subset K$ or $K \subset H$).

4. Prove that the following sets H of matrices are subgroups of $GL(2, \mathbb{R})$.

(a) (1 point) $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + c = 1,\ b + d = 1,\ ad - bc \neq 0,\ a, b, c, d \in \mathbb{R} \right\}$

(b) (1 point) $\left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a^2 + b^2 = 1,\ a, b \in \mathbb{R} \right\}$

5. (2 points) Let G be a group and let H be a nonempty subset of G such that whenever $x, y \in H$, we have $x(y^{-1}) \in H$. Prove that H is a subgroup of G.