**UH - Math 3330 - Dr. Heier - Spring 2019**
**HW 7**
**Due Wednesday, 03/06, at the beginning of class.**

**Your solution may be handwritten. Use regular sized sheets of paper, stapled together.**

**Do not forget to write your name on page 1.**

**1.** (1 point) Let $G = \{e, x_1, \ldots, x_{r-1}\}$ be an abelian group such that $r = \#G$ is an odd integer. Prove that

$$x_1 \cdot \ldots \cdot x_{r-1} = e.$$

Hint: Prove first that $x_1 \cdot \ldots \cdot x_{r-1}$ is its own inverse. Carefully explain your reasoning.

**2.**

(a) (1 point) Find the right cosets of the subgroup $H = \{(0,0),(1,0),(2,0)\}$ in $\mathbb{Z}_3 \times \mathbb{Z}_3$.

(b) (1 point) Find the right cosets of the subgroup $H = \{(0,0),(0,2)\}$ in $\mathbb{Z}_3 \times \mathbb{Z}_4$.

**3.** (2 points) Prove that $(\mathbb{Q},+)/(\mathbb{Z},+)$ is an infinite group such that each of its elements has finite order.

**4.** (1 point) Let $G$ be a group and let $H, K$ be two normal subgroups of $G$ with $H \cap K = \{e\}$. Prove that for $x \in H$ and $y \in K$, $xy = yx$ holds.

**5.** (2 points) Let $G$ be a group and let $N$ a normal subgroup of $G$. Let $H$ be a subgroup of $G$. Set $NH = \{nh \mid n \in N,\ h \in H\}$. Prove that $NH$ is a subgroup of $G$.

**6.** A subgroup $H$ of a group $G$ is characteristic if $\varphi(H) \subseteq H$ for every automorphism $\varphi$ of $G$.

(a) (1 point) Prove that every characteristic subgroup is normal.

(b) (1 point) Prove that the converse of (a) is false.