UH - Math 3330 - Dr. Heier - Spring 2019
HW 8
Due Wednesday, 04/03, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (1 point) Let G and H be groups. Prove that $G \times H$ and $H \times G$ are isomorphic groups.

2. (1 point) Let G be a group and $g \in G$ a fixed element. Prove that the map $\varphi : \mathbb{Z} \to G, n \mapsto g^n$ is a homomorphism.

3. (2 points) Let $G = \mathbb{Z}_8 \times \mathbb{Z}_2$. Find subgroups H and K of G such that H is isomorphic to K, but G/H and G/K are not isomorphic. Justify your answers carefully.

4. (1 point) If G is a group, let $Aut(G)$ denote the set of automorphisms of G. Show that $Aut(G)$ is a subgroup of $(S_G, \circ)$.

5. (2 points) Assume that $H \lhd K \lhd G$ and $H \lhd G$. Prove that K/H is a subgroup of G/H.

6. (1 point) Let $\varphi : (\mathbb{Z}_8, \oplus) \to (\mathbb{Z}_4, \oplus)$ be given by $\varphi(x) =$ remainder of x mod 4. Prove that φ is a surjective homomorphism. Determine $\ker(\varphi)$.

7. (2 points) Let $G = (\mathbb{C} \setminus \{0\}, \cdot)$, and let H be the subgroup $H = \{a + bi \mid a^2 + b^2 = 1\}$. Use the Fundamental Theorem to show that G/H is isomorphic to $(\mathbb{R}^+, \cdot)$