UH - Math 3330 - Dr. Heier - Spring 2019
HW 9
Due Wednesday, 04/10, at the beginning of class.

Your solution may be handwritten. Use regular sized sheets of paper, stapled together.

Do not forget to write your name on page 1.

1. (2 points) Let G and H be finite groups. Let $\varphi : G \to H$ be a surjective homomorphism. Prove that $|H|$ divides $|G|$.

2. (2 points) Let $\varphi : G \to K$ be a surjective homomorphism. Let $J \lhd K$. Prove that there exists a normal subgroup H of G such that G/H is isomorphic to K/J.

3. Find, up to isomorphism, all abelian groups of order

(a) (1 point) 324,

(b) (1 point) 900.

4. (2 points) Let G be an abelian group of order p^n, where p is prime. An element $x \in G$ is said to be of maximal order if $\text{ord}(x) \geq \text{ord}(y)$ for all $y \in G$. Prove that the only subgroup of G that contains all the elements of maximal order is G itself.

5. Let R be a ring and $a, b \in R$. Prove that

(a) (1 point) $a \cdot 0 = 0 = 0 \cdot a$,

(b) (1 point) $(-a) \cdot b = a \cdot (-b) = -(a \cdot b)$.