UH - Math 4377/6308 - Dr. Heier - Spring 2020
HW 5
Due 02/20, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Section 1.6, Problem 1 (Just say true or false, no further explanation necessary.)

2. (1 point) Section 1.6, Problem 8

3. (1 point) Let V be a vector space over $\mathbb{R}$. Let $v, w \in V$. Prove that if $\{v - w, v + w\}$ is linearly independent, then $\{v, w\}$ is linearly independent.

4. For each of the following subspaces of $\mathbb{R}^5$, find a basis.

(a) (0.5 points) $W_1 = \{(a, b, c, d, e) \in \mathbb{R}^5 : a - b + c - d + e = 0\}$

(b) (0.5 points) $W_2 = \{(a, b, c, d, e) \in \mathbb{R}^5 : a = c \text{ and } a + b + c = d \text{ and } c + d + e = 0\}$

The next two problems give examples of how the Replacement Theorem 1.10 discussed in class works in concrete situations.

5. Let $G = \{(1, -1, 3, 2), (1, 1, 0, 1), (1, -2, 1, 7), (0, 2, 1, 1)\}$. Let $L = \{(1, -1, -1, 0)\}$.

(a) (1 point) Show that G spans $\mathbb{R}^4$. (Since it has 4 elements, G is then automatically a basis, but we are only interested in the spanning property.)

(b) (1 point) Find a subset $H \subset G$ of cardinality 3 such that $H \cup L$ spans $\mathbb{R}^4$. Prove the spanning property with an explicit computation.

6. (2 points) Let $L = \{(1, 2, 1, 3), (0, 0, 1, 1)\}$. Let $G = \{v_1 = (1, 2, -2, 0),\ v_2 = (1, 0, 0, -1),\ v_3 = (0, 1, 1, 1),\ v_4 = (1, 2, 2, 4)\}$. You can assume without proof that G spans $\mathbb{R}^4$. Find two vectors in G that can be replaced by the two elements of L in such a way that the spanning property is preserved.

7. (2 points) Section 1.6, Problem 30