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A FUNCTIONAL CALCULUS FOR PAIRS OF COMMUTING POLYNOMIALLY BOUNDED OPERATORS

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ABSTRACT. A functional calculus valid for the class of absolutely continuous pairs of commuting polynomially bounded operators is defined.

1. INTRODUCTION

Let $\mathcal{H}$ be a separable infinite dimensional, complex Hilbert space, and denote by $\mathcal{L}(\mathcal{H})$ the algebra of bounded linear operators on $\mathcal{H}$. In [11] Sz-Nagy and Foiaş developed an $\mathbf{H}^\infty$-functional calculus for any completely nonunitary contraction $T \in \mathcal{L}(\mathcal{H})$ (in fact, this functional calculus is valid for the larger class $ACC(\mathcal{H})$ consisting of the absolutely continuous contractions on $\mathcal{H}$). A two-variable version of the Nagy-Foiaş functional calculus was constructed in [5], valid for commuting pairs of completely nonunitary contractions. This functional calculus can be extended to the larger class $ACC^{(2)}(\mathcal{H})$ consisting of absolutely continuous pairs of commuting contractions on $\mathcal{H}$ [9]. These functional calculi use the dilation theory that exists in either of this two cases, cf., [11], Chapter I. For other functional calculi see [2], [5], [6], [7], [8], [10].

In Section 3, we present an $\mathbf{H}^\infty$-functional calculus for commuting operators T_1 and T_2 satisfying the property that for a given $M \geq 1$ and for all (complex) two variable polynomials p we have $\|p(T_1, T_2)\| \leq M\|p\|_\infty$. The functional calculus defined in Section 3 is not based on any dilation theory. For another approach to a functional calculus for certain commuting polynomially bounded operators see [2], [6].

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2. Preliminaries

Let $\mathbb{D}$ denote the open disc in the complex plane $\mathbb{C}$ and $\mathbb{T} = \partial\mathbb{D}$ the unit circle. A complex, Borel measure ν on $\mathbb{T}^2$ is called an *annihilating measure* for $A(\mathbb{D}^2)$, the algebra of analytic functions on $\mathbb{D}^2$ and continuous on $\mathbb{D}^{2-}$, if for all $u \in A(\mathbb{D}^2)$

$$\int_{\mathbb{T}^2} u\, d\nu = 0. \tag{2.1}$$

The set of all annihilating measures of $A(\mathbb{D}^2)$ will be denoted by $A^\perp$ [5].

Let $\mathcal{U}$ be a measurable space, and denote by $M(\mathcal{U})$ the set of all (finite) complex Borel measures defined on $\mathcal{U}$. A set $\Gamma \subseteq M(\mathcal{U})$ is called a *band* [5] on $\mathcal{M}$ if it is closed under absolute continuity, i.e., if $\nu \in \Gamma$ and $\mu \in M(\mathcal{U})$ is absolutely continuous with respect to ν (notation: $\mu \ll \nu$), then $\mu \in \Gamma$, and it is also closed under infinite sums, i.e., if $\{\nu_n\}$ is a sequence in Γ with $\sum_{n=1}^{\infty} |\nu_n| < \infty$, then $\sum_{n=1}^{\infty} \nu_n \in \Gamma$. If Γ is a band on $\mathcal{U}$, we denote by $\Gamma^\perp$ the set of all complex measures on $\mathcal{U}$ singular with respect to every measure in Γ. It is easy to see that $\Gamma^\perp$ is a band and the intersection of two bands is a band. Note that $A^\perp$ is a band on $\mathbb{T}^2$.

We then define the following two bands on $\mathbb{T}^2$, cf., [5]. Let Γ_1 be the set of all $\eta \in B(\mathbb{T}^2)$ carried by sets of the form $E \times \mathbb{T}$, where E is a Borel set in $\mathbb{T}$ such that $m_1(E) = 0$, where m_1 is the normalized Lebesgue measure on $\mathbb{T}$. Let Γ_2 be the set of all $\eta \in B(\mathbb{T}^2)$ carried by sets of the form $\mathbb{T} \times F$, where F is a Borel set in $\mathbb{T}$ such that $m_1(F) = 0$. Then we write

$$\Gamma = A^\perp \cap \Gamma_1^\perp \cap \Gamma_2^\perp.$$

Lemma 2.1. ([5], Lemma 2.1) *Let $\nu \in \Gamma$, and let $\{u_n\}$ be a bounded sequence in $A(\mathbb{D}^2)$. Then $u_n \overset{n}{\to} 0$ pointwise on $\mathbb{D}^2$ if and only if $u_n \overset{n}{\to} 0$ in the weak* topology of $\mathbf{L}^\infty(\mathbb{T}^2, |\nu|)$.*

Let $\nu \in \Gamma$ be a measure, and define the algebra $\mathbf{H}^\infty(\nu + m_2)$ to be the weak* closure of the bidisk algebra $A(\mathbb{D}^2)$ in $\mathbf{L}^\infty(\mathbb{T}^2, \nu + m_2)$ (the space of essentially-bounded functions on $\mathbb{T}^2$ with respect to the measure $\nu + m_2$). The Poisson integral

$$P[u](\zeta) = \int_{\mathbb{T}^2} P(\zeta, w) u(w) dm_2(w), \qquad \zeta \in \mathbb{D}^2,$$

can be used to define an isometric isomorphism between $\mathbf{H}^\infty(\nu+m_2)$ and $\mathbf{H}^\infty(\mathbb{D}^2)$, the algebra of bounded analytic functions on $\mathbb{D}^2$. We define a map $\Theta : \mathbf{H}^\infty(\mathbb{D}^2) \to \mathbf{H}^\infty(\nu + m_2)$ by extending the identity map from $A(\mathbb{D}^2)$ onto itself in such a way as to make Θ a weak*-continuous algebra isomorphism. For u in $A(\mathbb{D}^2)$ we define $\Theta(u) = u$. For u in $\mathbf{H}^\infty(\mathbb{D}^2)$ we approximate u in the weak*-topology by a bounded sequence of polynomials $\{p_n\}$. For each $\zeta \in \mathbb{D}^2$, $p_n(\zeta) \xrightarrow{n} u(\zeta)$. Hence, Lemma 2.1 implies that there exists a function $\hat{u}$ in $\mathbf{H}^\infty(\nu + m_2)$, which is the weak*-limit of the sequence $\{p_n\}$ in $\mathbf{H}^\infty(\nu + m_2)$. For $u \in \mathbf{H}^\infty(\mathbb{D}^2)$ we then define $\Theta(u) = \hat{u}$. We clearly have that Θ is multiplicative, and, since sequences are enough to determine weak*-continuity (cf., [4]), the following identification results [5]:

Proposition 2.2. *If $\nu \in \Gamma$, there exists an isometric weak*-continuous algebra isomorphism Θ from $\mathbf{H}^\infty(\nu + m_2)$ onto $\mathbf{H}^\infty(\mathbb{D}^2)$.*

Let T_1 and T_2 be commuting operators in $\mathcal{L}(\mathcal{H})$. (The class of commuting operators in $\mathcal{L}(\mathcal{H})$ is denoted by $\mathcal{L}(\mathcal{H})^{(2)}_{comm}$.) We then consider the dual algebra $\mathcal{A}_{T_1,T_2}$ generated by the pair (T_1, T_2) in $\mathcal{L}(\mathcal{H})^{(2)}_{comm}$. The predual of $\mathcal{A}_{T_1,T_2}$ will be denoted by $\mathcal{Q}_{T_1,T_2}$. Let us also consider $\mathbf{H}^\infty(\mathbb{D}^2)$ with predual

$$\mathbf{H}^\infty_*(\mathbb{D}^2) = \mathbf{L}^1(\mathbb{T}^2)/\mathbf{L}^1_0(\mathbb{T}^2)$$

($\mathbf{L}^1(\mathbb{T}^2)$ is the class of all integrable functions on $\mathbb{T}^2$ with respect to the normalized measure m_2 on $\mathbb{T}^2$ and $\mathbf{L}^1_0(\mathbb{T}^2)$ is the set of all $\mathbf{L}^1(\mathbb{T}^2)$-functions with vanishing negative multi-index coefficients), and let $\Phi : \mathbf{H}^\infty(\mathbb{D}^2) \to \mathcal{L}(\mathcal{H})$ be a weak*-continuous unital homomorphism. Then, interesting results regarding the invariant subspaces of Range Φ can be obtained by using dual algebra techniques [1], [4]. Therefore, we are interested in pairs $(T_1, T_2) \in \mathcal{L}(\mathcal{H})^{(2)}_{comm}$ for which a "good" $\mathbf{H}^\infty(\mathbb{D}^2)$-functional calculus exists.

3. A FUNCTIONAL CALCULUS

We say that a single operator T in $\mathcal{L}(\mathcal{H})$ is *polynomially bounded* if there exists a constant $M \geq 1$ such that for any (complex) one variable polynomial we have

$$\|p(T)\| \leq M\|p\|_\infty,$$

where by $\|p\|_\infty$ we understand $\|p\|_\infty = \sup_{z \in \mathbb{D}^-} |p(z)|$. We then write $PB^M(\mathcal{H})$ for the class of such operators and, in general,

$$PB(\mathcal{H}) = \bigcup_{M \geq 1} PB^M(\mathcal{H})$$

denotes the class of all polynomially bounded operators on $\mathcal{H}$.

Definition 1. Let (T_1, T_2) be in $\mathcal{L}(\mathcal{H})^{(2)}_{comm}$. We say that the pair (T_1, T_2) is *2-polynomially bounded* if there exists a constant $M \geq 1$ such that

$$\|p(T_1, T_2)\| \leq M\|p\|_\infty,$$

for all (complex) two variable polynomials and we write

$$(T_1, T_2) \in PB^{(2), M}(\mathcal{H}),$$

where by $\|p\|_\infty$ we understand $\|p\|_\infty = \sup_{(z_1,z_2)\in\mathbb{D}^2} |p(z_1, z_2)|$.

The set of all 2-polynomially bounded operators will be denoted by

$$PB^{(2)}(\mathcal{H}) = \bigcup_{M\geq 1} PB^{(2), M}(\mathcal{H}).$$

Remark. If (T_1, T_2) is in $PB^{(2), M}(\mathcal{H})$ then T_1 and T_2 are in $PB^M(\mathcal{H})$.

For $(T_1, T_2) \in PB^{(2), M}(\mathcal{H})$ and p a two-variable polynomial, the mapping $p \mapsto p(T_1, T_2)$ is a norm continuous algebraic homomorphism, and it extends to a continuous unital algebra homomorphism on $A(\mathbb{D}^2)$, the bidisk algebra. Let

$$\Psi_{T_1,T_2} : A(\mathbb{D}^2) \to \mathcal{A}_{T_1,T_2}$$

satisfying

$$\Psi_{T_1,T_2}(1) = I_{\mathcal{H}}, \quad \Psi_{T_1,T_2}(z_1) = T_1, \quad \Psi_{T_1,T_2}(z_2) = T_2.$$

For all x and y in $\mathcal{H}$ define the following bounded linear functional $\Psi^{x,y}_{T_1,T_2} : A(\mathbb{D}^2) \to \mathbb{C}$ by

$$\Psi^{x,y}_{T_1,T_2}(u) = \langle u(T_1, T_2)x, y\rangle.$$

We may extend $\Psi^{x,y}_{T_1,T_2}$ to be a bounded linear functional of the same norm on $C(\mathbb{T}^2)$. In fact, the algebra $A(\mathbb{D}^2)$ can be identified with the algebra of bounded analytic functions on $\mathbb{D}^2$ that have a continuous extension to $\mathbb{T}^2$. By Riesz representation theorem for $C(\mathbb{T}^2)$ there exists a regular complex Borel measure supported on $\mathbb{T}^2$ such that

$$\Psi^{x,y}_{T_1,T_2}(u) = \langle u(T_1, T_2)x, y\rangle = \int_{\mathbb{T}^2} u \, d\mu_{x,y}, \quad u \in A(\mathbb{D}^2),$$

and clearly $|\mu_{x,y}| \leq M\|x\| \, \|y\|$. Such a family $\{\mu_{x,y}\}_{x,y\in\mathcal{H}}$ of Borel measures supported on $\mathbb{T}^2$ will be called family of *elementary measures* for (T_1, T_2).

For x and y in $\mathcal{H}$ let $\mu_{x,y}$ and $\nu_{x,y}$ be elementary measures for $(T_1, T_2) \in PB^{(2)}(\mathcal{H})$. Then

$$\mu_{x,y} - \nu_{x,y} \in A^\perp,$$

and, therefore, for every $u \in A(\mathbb{D}^2)$ we can write

$$\langle u(T_1, T_2)x, y\rangle = \int_{\mathbb{T}^2} u \, d\mu_{x,y} = \int_{\mathbb{T}^2} u \, d\nu_{x,y}. \tag{3.1}$$

Using the notation introduced in Section 2, we have

Definition 2. A pair (T_1, T_2) in $PB^{(2)}(\mathcal{H})$ is said to be *absolutely continuous* if for every pair (x, y) in $\mathcal{H}$ there exists an elementary measure $\mu_{x,y}$ for (T_1, T_2) such that for some ν in Γ, $\mu_{x,y} \ll \nu + m_2$,

We then write $ACPB^{(2),M}(\mathcal{H})$ for the class of all absolutely continuous pairs of operators (T_1, T_2) in $PB^{(2),M}(\mathcal{H})$ and

$$ACPB^{(2)}(\mathcal{H}) = \bigcup_{M \geq 1} ACPB^{(2),M}(\mathcal{H}).$$

Then the following functional calculus is valid:

Theorem 3.1. *If $(T_1, T_2) \in ACPB^{(2)}(\mathcal{H})$, then there exists a unique unital algebra homomorphism*

$$\Phi_{T_1,T_2} : \mathbf{H}^\infty(\mathbb{D}^2) \to \mathcal{A}_{T_1,T_2}$$

with the following properties:

1. *For $j = 1, 2$, $\Phi_{T_1,T_2}(z_j) = T_j$;*
2. $\Phi_{T_1,T_2}|_{A(\mathbb{D}^2)} = \Psi(T_1, T_2)$;
3. *If (T_1, T_2) is in $ACPB^{(2),M}(\mathcal{H})$, then for all $u \in \mathbf{H}^\infty(\mathbb{D}^2)$*
$$\| \Phi_{T_1,T_2}(u) \| \leq M \| u \|_\infty;$$
4. *Φ_{T_1,T_2} is weak*-continuous (i.e., continuous when both spaces $\mathbf{H}^\infty(\mathbb{D}^2)$ and $\mathcal{A}_{T_1,T_2}$ are given their weak*-topologies);*
5. *The range of Φ_{T_1,T_2} is weak*-dense in $\mathcal{A}_{T_1,T_2}$;*
6. *There is a bounded, linear, one-to-one map*
$$\phi_{T_1,T_2} : \mathcal{Q}_{T_1,T_2} \to \mathbf{L}^1(\mathbb{T}^2)/\mathbf{L}^1_0(\mathbb{T}^2)$$
with $\phi^*_{T_1,T_2} = \Phi_{T_1,T_2}$;
7. *If Φ_{T_1,T_2} is bounded below, then the range of Φ_{T_1,T_2} is $\mathcal{A}_{T_1,T_2}$ and Φ is an invertible algebra isomorphism of $\mathbf{H}^\infty(\mathbb{D}^2)$ onto $\mathcal{A}_{T_1,T_2}$; in this case, ϕ_{T_1,T_2} is invertible linear isomorphism of the space $\mathcal{Q}_{T_1,T_2}$ onto the quotient space $\mathbf{L}^1(\mathbb{T}^2)/\mathbf{L}^1_0(\mathbb{T}^2)$;*
8. *If $\sigma_T(T_1, T_2) \subset \mathbb{D}^2$ then $\Phi_{T_1,T_2}(u) = u(T_1, T_2)$ where $u(T_1, T_2)$ is given by the Taylor functional calculus* [12].
9. *If T_1 and T_2 are completely nonunitary contractions (or (T_1, T_2) belongs to $ACC^{(2)}(\mathcal{H})$) then the above functional calculus coincides with the one defined in* [5], [9].

PROOF. Let $(T_1, T_2) \in ACPB^{(2)}(\mathcal{H})$, and for any pair of vectors x and y in $\mathcal{H}$ let us consider $\mu_{x,y}$ an elementary measure for (T_1, T_2), i.e., for some $\nu \in \Gamma$ we have that $\mu_{x,y} \ll \nu + m_2$. For any $u \in \mathbf{H}^\infty(\mathbb{D}^2) \cong \mathbf{H}^\infty(\nu + m_2)$ let $\{u_n\}$ in $A(\mathbb{D}^2)$ be weak*-convergent to u, and then we may write

$$\begin{aligned}\int_{\mathbb{T}^2} u\, d\mu_{x,y} &= \int_{\mathbb{T}^2} u \frac{d\mu_{x,y}}{d(\nu+m_2)} d(\nu+m_2)\\ &= \lim_{n\to\infty} \int_{\mathbb{T}^2} u_n \frac{d\mu_{x,y}}{d(\nu+m_2)} d(\nu+m_2),\end{aligned}$$

where, by the Radon-Nikodym theorem,

$$\frac{d\mu_{x,y}}{d(\nu+m_2)} \in \mathbf{L}^1(\mathbb{T}^2, \nu+m_2),$$

and so

$$\int_{\mathbb{T}^2} u\, d\mu_{x,y} = \lim_{n\to\infty} \int_{\mathbb{T}^2} u_n\, d\mu_{x,y} = \lim_{n\to\infty} \langle u_n(T_1,T_2)x, y\rangle.$$

Therefore, for $(T_1, T_2) \in ACPB^{(2)}(\mathcal{H})$ and for any x and y in $\mathcal{H}$ we then define

$$\langle u(T_1,T_2)x, y\rangle = \int_{\mathbb{T}^2} u\, d\mu_{x,y}, \qquad u \in \mathbf{H}^\infty(\mathbb{D}^2). \tag{3.2}$$

Using (3.1), one can easily remark that (3.2) does not depend on the choice of $\mu_{x,y}$. Since

$$\left|\int_{\mathbb{T}^2} u\, d\mu_{x,y}\right| \le \| u \|_\infty \| x \| \| y \|,$$

we have then defined a sesquilinear bounded map, which yields a well-defined operator $u(T_1, T_2)$ in $\mathcal{A}_{T_1,T_2}$. Define

$$\Phi_{T_1,T_2}(u) = u(T_1, T_2),$$

where $u(T_1, T_2)$ is given by (3.2), and then it is immediate that Φ is linear and satisfies (1), (2), and (3). That Φ_{T_1,T_2} is multiplicative is a consequence of the fact that Φ_{T_1,T_2} is multiplicative on polynomials. For any pair of vectors x and y in $\mathcal{H}$, let us consider $\mu_{x,y}$ an elementary measure for (T_1, T_2). Let u be an arbitrary function in $\mathbf{H}^\infty(\mathbb{D}^2) \cong \mathbf{H}^\infty(\nu + m_2)$ (*see Proposition 2.2 for the isomorphism*). Using the computation before (3.2) for some sequence $\{u_n\}$ in $\mathbf{H}^\infty(\mathbb{D}^2)$ weak*-convergent to u, we obtain

$$\langle u(T_1,T_2)x, y\rangle = \int_{\mathbb{T}^2} u\, d\mu_{x,y} = \lim_{n\to\infty} \int_{\mathbb{T}^2} u_n\, d\mu_{x,y} = \lim_{n\to\infty} \langle u_n(T_1,T_2)x, y\rangle.$$

Thus, the sequence $\{u_n(T_1, T_2)\}$ *converges to* $u(T_1, T_2)$ *in the (WOT)-topology.* Since on bounded sets the weak-operator topology and the weak*-topology coincide, we have obtained (4) (sequences are enough to determine weak*-continuity [4]). By definition, the polynomials are weak*-dense in $\mathcal{A}_{T_1,T_2}$, and thus, we obtain (5). Moreover, since a linear map between Banach spaces is weak*-continuous if, and only if it is the adjoint of a bounded map [4], then (6) becomes obvious, and (7) as well. If $\sigma_T(T_1, T_2) \subset \mathbb{D}^2$ then Φ_{T_1,T_2} coincides with the Taylor analytic functional calculus because the two representations coincide on polynomials (all we have to remark is that for any $u \in \mathbf{H}^\infty(\mathbb{D}^2)$ one can write the Cauchy representation for $u(T_1, T_2)$ on some bidisc $D = D_1 \times D_2$ satisfying $\mathbb{D}^2 \supset D^- \supset D \supset \sigma_T(T_1, T_2)$, and then use a uniform approximation by polynomials), and this concludes (8). If T_1 and T_2 are commuting completely nonunitary contractions or $(T_1, T_2) \in ACC^{(2)}(\mathcal{H})$, then the above functional calculus coincides with the one defined in [5] and [9] because of a similar reason (these functional calculi are weak*-continuous and coincide on polynomials), and (9) follows. We remark that, in this later case, i.e., for pairs of commuting completely nonunitary contractions, a family $\{\mu_{x,y}\}_{x,y\in\mathcal{H}}$ of elementary measures for (T_1, T_2) can be explicitely defined with respect to the amalgamation given by a unitary dilation of the pair (T_1, T_2) of commuting completely nonunitary contractions (cf., [3], [5]). □

Example 3.2. Let (T_1, T_2) be a pair of commuting completely nonunitary contractions and let S be an invertible operator which commutes with T_2. Then, for some $M \leq \|S^{-1}\| \cdot \|S\|$ the pair (ST_1S^{-1}, T_2) is in $ACPB^{(2), M}(\mathcal{H})$.

PROOF. Obviously, we have that $(ST_1S^{-1}, T_2) \in PB^{(2), M}(\mathcal{H})$, for some $M \leq \|S^{-1}\| \cdot \|S\|$. For T_1 and T_2 as above and for any vectors x and y in $\mathcal{H}$ we have the following representation:

$$\langle u(T_1, T_2)x, y\rangle = \int_{\mathbb{T}^2} u\, d\mu_{x,y}, \qquad u \in \mathbf{H}^\infty(\mathbb{D}^2),$$

where, for some $\nu \in \Gamma$, $\mu_{x,y} \ll \nu + m_2$. In particular, we have that

$$\langle u(T_1, T_2)S^{-1}x, S^*y\rangle = \int_{\mathbb{T}^2} u\, d\mu_{S^{-1}x, S^*y},$$

where for some $\hat{\nu} \in \Gamma$, $\mu_{S^{-1}x,S^*y} \ll \hat{\nu} + m_2$. Let us denote $\hat{\mu}_{x,y} = \mu_{S^{-1}x,S^*y}$; then

$$\begin{aligned} \langle u(ST_1S^{-1}, T_2)x, y\rangle &= \langle u(T_1, T_2)S^{-1}x, S^*y\rangle \\ &= \int_{\mathbb{T}^2} u\, d\mu_{S^{-1}x,S^*y} = \int_{\mathbb{T}^2} u\, d\hat{\mu}_{x,y}. \end{aligned}$$

Therefore, we have that $(ST_1S^{-1}, T_2) \in ACPB^{(2), M}(\mathcal{H})$. □

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