

**Math 1311**  
**Section 6.4**  
**Equations of Change: Linear and Exponential Functions**

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**Definition:** An equation of the form  $\frac{df}{dx} = \text{Right-hand side}$  is called an equation of change, also known as differential equation.

**Equations of Change and Linear Functions**

The equation of change  $\frac{df}{dx} = m$ , where  $m$  is a constant, says that  $f$  has a constant rate of change  $m$  and hence that  $f$  is a linear function with slope  $m$ . That is,  $f = mx + b$ .

An initial condition is needed to determine the value of  $b$ .

**Equations of Change and Exponential Functions**

The equation of change  $\frac{df}{dx} = rf$ , where  $r$  is a constant, says that  $f$  has a constant proportional (and hence percentage) rate of change and is therefore an exponential function. The exponential growth rate for  $f$  is  $r$ , so the growth (or decay) factor is  $e^r$ . That is,

$$f = Pe^{rx}$$

Or

$$f = P \times (e^r)^x$$

where  $P$  is the initial value of  $f$ .

**Example 1:** On Mars, a falling object satisfies the equation of change  $\frac{dV}{dt} = 12.16$ , where  $V$  is downward velocity in feet per second and  $t$  is time in seconds.

- a) What is the value of acceleration due to gravity on Mars?
- b) Suppose an astronaut stands atop a cliff on Mars and throws a rock downward with an initial velocity of 8 feet per second. What is the velocity of the rock 3 seconds after release?

**Example 2:** You open an account by investing \$250 with a financial institution that advertises an APR of 5.75%, with continuous compounding.

- a) Find an exponential formula for the balance in your account as a function of time. In your answer, give both the standard form and the exponential function.
- b) What account balance would you expect 5 years after your initial investment? Answer this question using both of the forms you found in part a). Which do you think gives more accurate answer? Why?

**Example 3:** What is the common mathematical term for an equation of change?

**Example 4:** If  $f$  satisfies the equation of change  $\frac{df}{dx} = m$ , what kind of function is  $f$ ?

**Example 5:** If  $f$  satisfies the equation of change  $\frac{df}{dx} = 3$ , then  $f$  is a linear function. What is the slope of  $f$ ?

**Example 6:** If  $f$  satisfies the equation of change  $\frac{df}{dx} = cf$ , what kind of function is  $f$ ?

**Example 7:** If  $f$  satisfies the equation of change  $\frac{df}{dx} = 5f$ , then  $f$  is an exponential function and hence can be written as  $f = Ae^{ct}$ . What is the value of  $c$ ?

**Example 8:** Solve the equation of change  $\frac{df}{dx} = 5$  if the initial value of  $f$  is 3.

**Example 8:** Solve the equation of change  $\frac{df}{dx} = 5f$  if the initial value of  $f$  is 2. Use the alternative form for exponential function.