Math 1311
Section 6.4
Equations of Change: Linear and Exponential Functions

Definition: An equation of the form $\frac{df}{dx} = Right - hand\ side$ is called an equation of change, also known as differential equation.

Equations of Change and Linear Functions

The equation of change $\frac{df}{dx} = m$, where m is a constant, says that f has a constant rate of change m and hence that f is a linear function with slope m. That is, $f = mx + b$.

An initial condition is needed to determine the value of b.

Equations of Change and Exponential Functions

The equation of change $\frac{df}{dx} = rf$, where r is a constant, says that f has a constant proportional (and hence percentage) rate of change and is therefore an exponential function. The exponential growth rate for f is r, so the growth (or decay) factor is e^r. That is,

$$f = Pe^{rx}$$

Or

$$f = P \times (e^r)^x$$

where P is the initial value of f.

Example 1: On Mars, a falling object satisfies the equation of change $\frac{dV}{dt} = 12.16$, where V is downward velocity in feet per second and t is time in seconds.

a) What is the value of acceleration due to gravity on Mars?
b) Suppose an astronaut stands atop a cliff on Mars and throws a rock downward with an initial velocity of 8 feet per second. What is the velocity of the rock 3 seconds after release?

(a) $a = \frac{dV}{dt} = 12.16$ ft per sec^2 $\qquad \frac{\text{ft per second}}{\text{second}}$

(b) $V(t) = 12.16t + b \leftarrow$ initial value

$V(t) = 12.16t + 8$

$V(3) = 12.16(3) + 8 = 44.48$ ft/sec

Example 2: You open an account by investing $250 with a financial institution that advertises an APR of 5.75%, with continuous compounding.

a) Find an exponential formula for the balance in your account as a function of time. In your answer, give both the standard form and the exponential function.
b) What account balance would you expect 5 years after your initial investment? Answer this question using both of the forms you found in part a). Which do you think gives more accurate answer? Why?

(a) $r = .0575$ $P = 250$

$B(t) = 250\, e^{.0575t}$

(b) $B(5) = 250\, e^{.0575(5)}$

$\$ = 333.27$

$B(t) = 250 \times 1.0592^{t}$

$B(5) = 250 \times (1.0592)^{5}$

$\$ 333.30$

Example 3: What is the common mathematical term for an equation of change?

diff. eq.

Example 4: If f satisfies the equation of change $\frac{df}{dx} = m$, what kind of function is f?

linear

Example 5: If f satisfies the equation of change $\frac{df}{dx} = 3$, then f is a linear function. What is the slope of f?

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Example 6: If f satisfies the equation of change $\frac{df}{dx} = cf$, what kind of function is f?

expon.

Example 7: If f satisfies the equation of change $\frac{df}{dx} = 5f$, then f is an exponential function and hence can be written as $f = Ae^{ct}$. What is the value of c?

$c = 5$

Example 8: Solve the equation of change $\frac{df}{dx} = 5$ if the initial value of f is 3.

$f(x) = 5x + 3$

Example 8: Solve the equation of change $\frac{df}{dx} = 5f$ if the initial value of f is 2. Use the alternative form for exponential function.

$f(x) = 2e^{5x}$