Math 1311
Section 6.5
Equations of Change: Graphical Solutions

Equilibrium or Steady-State Solutions

Equilibrium or steady-state solutions of an equation of change occur where the rate of change is 0. We get them by setting the rate of change equal to 0 and solving. In many cases they show long-term behavior.

Example 1: A skydiver falls subject to the equation of change $\frac{dV}{dx} = 32 - 0.1818V$, where velocity V is measured in feet per second and time t in seconds.

Find the equilibrium solutions of the equation of change and investigate their physical meaning.

$$32 - .1818V = 0$$
$$-32 \qquad\qquad -32$$
$$\frac{-.1818V}{-.1818} = \frac{-32}{-.1818}$$

Velocity $\rightarrow$ $\boxed{V = 176 \text{ ft per sec.}}$

Example 2: In California in the 1930s and 1940s, a large part of the fishing related industry was based on the catch of Pacific sardines. Studies have shown that the sardine population grows approximately logistically. The logistic equation has been determined experimentally to be $\frac{dN}{dt} = 0.338N(1 - \frac{N}{2.4})$, where N is measured in millions of ton of fish and t is measured in years.

Find the equilibrium solutions and give a physical interpretation of their meaning.

$$\underbrace{.338N\left(1 - \frac{N}{2.4}\right)}_{Y_1} = \underbrace{0}_{Y_2}$$

$$\boxed{N = 2.4 \text{ million of ton of fish}}$$

Example 3: What is an equilibrium solution of an equation of change?

$\frac{df}{dx} = 0$ rate of change $= 0$

Example 4: If $f = 10$ is an equilibrium solution of an equation of change involving $\frac{df}{dx}$, what is the value of $\frac{df}{dx}$ when f is 10?

$\frac{df}{dx} = 0$ when $f = 10$

Example 5: Find an equilibrium solution of $\frac{df}{dx} = 2f - 6$.

$2f - 6 = 0$

$2f = 6$

$\boxed{f = 3}$

Example 6: Water flows in a tank, and a certain part of it drains out through a valve. The volume v in cubic feet of water in the tank at time t satisfies the equation $\frac{dv}{dt} = 5 - \frac{v}{3}$. If the process continues for a long time, how much water will be in the tank?

$5 - \frac{v}{3} = 0$

$-5 \qquad -5$

$(-\cancel{3}) \cdot -\frac{v}{\cancel{3}} = -5(-3)$

$\boxed{v = 15 \text{ ft}^3}$

Example 7: At some time in the process described in question ~~4~~ 6 there are 8 cubic feet of water in the tank. Is the volume increasing or decreasing?

$8 < 15$

Using the Equation of Change to Graph Functions

To make a hand-drawn sketch of the graph of f versus x from the equation of change for f, first draw horizontal lines representing any obvious equilibrium solutions. Next, use the calculator to graph $\frac{df}{dx}$ versus f. (If the equation of change is of the form

$$\frac{df}{dx} = \text{Right- hand side}$$

you graph the right-hand side.)

1. Where the graph of $\frac{df}{dx}$ versus f crosses the horizontal axis, we have $\frac{df}{dx} = 0$, so f does not change. This equilibrium solution for f.
2. When the graph $\frac{df}{dx}$ versus f is above the horizontal axis, the graph of f versus x is increasing.
3. When the graph $\frac{df}{dx}$ versus f is below the horizontal axis, the graph of f versus x is decreasing.