Lecture 27

11.7 Power Series 11.8 Differentiation and Integration of Power Series

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Power Series



As the Limit

f can be viewed as the limit of a sequence of polynomials: $f(x) = \lim_{n \to \infty} p_n(x),$ where $p_n(x) = 1 + x + x^2 + x^3 + \dots + x^n$

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$$\sum_{k=0}^{\infty} x^{k} = 1 + x + x^{2} + x^{3} + \dots \begin{cases} \frac{1}{1-x}, & \text{if } |x| < 1, \\ & \text{diverges,} & \text{if } |x| \ge 1. \end{cases}$$

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Define a function
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 on the interval $(-1, 1)$
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Geometric Series: $\sum_{k=0}^{\infty} x^k$

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Power Series Diff and Integ Geometric Series Radius of Convergence

Variations on the Geometric Series (I)

Closed forms for many power series can be found by relating the series to the geometric series

Examples



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Power Series Diff and Integ Geometric Series

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Power Series Diff and Integ

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$$= x \sum_{k=0}^{\infty} \left(\frac{x^2}{3}\right)^k = \frac{x}{1 - (x^2/3)} = \frac{3x}{3 - x^2} \quad \text{for } |x^2/3| < 1.$$

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Radius of Convergence



The radius of convergence of a power series can usually be found by applying the ratio test. In some cases the root test is easier.

Example



Thus the series converges absolutely when |x| < 1 and diverges when |x| > 1.



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Example

$$f(x) = \sum_{k=1}^{\infty} k^2 x^k = x + 4x^2 + 9x^3 + \cdots$$

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$$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{(k+1)^2 x^{k+1}}{k^2 x^k}\right|$$
$$= \frac{(k+1)^2}{k^2} |x| \to |x| \text{ as } k \to \infty$$

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Ratio Test :
$$\left| \frac{a_{k+1}}{a_k} \right| = \left| \frac{x^{k+1}/(k+1)!}{x^k/k!} \right|$$
$$= \frac{k!}{(k+1)!} \left| \frac{x^{k+1}}{x^k} \right| = \frac{1}{k+1} |x| \to 0 < 1 \quad \text{for all } x$$

Thus the series converges absolutely for all m x



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Thus the series converges absolutely for all x.

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The radius of convergence of a power series can usually be found by applying the ratio test. In some cases the root test is easier.

Example

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} x^k = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \cdots = e^{-x}$$

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$$f(x) = \sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^{k^2} x^k = 2x + (3/2)^4 x^2 + (4/3)^9 x^3 + \cdots$$

Ratio Test : $(|a_k|)^{\frac{1}{k}} = \left(\left(\frac{k+1}{k}\right)^{k^2} |x|^k\right)^{\frac{1}{k}} = \left(\frac{k+1}{k}\right)^k |x|$
$$= \left(1 + \frac{1}{k}\right)^k |x| \to e|x| < 1 \quad \text{if } |x| < 1/e$$

Thus the series converges absolutely when $|x| < 1/e$ and diverges when $|x| > 1/e$.



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Thus the series converges absolutely when |x|<1/e and diverges when |x|>1/e.



For a series with radius of convergence r, the interval of convergence can be [-r, r], (-r, r], [-r, r), or (-r, r).

Example

In general, the behavior of a power series at -r and at r is not predictable. For example, the series

$$\sum x^k, \quad \sum \frac{(-1)^k}{k} x^k, \quad \sum \frac{1}{k} x^k, \quad \sum \frac{1}{k^2} x^k$$



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$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} x^{k}$$
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Theorem

Let $f(x) = \sum a_k x^k$ be a power series with a nonzero radius of convergence r. Then

 $f'(x) = \sum a_k k x^{k-1} \quad \text{for } |x| < r$ $\int f(x) \, dx = \sum \frac{a_k}{k+1} x^{k+1} + C \quad \text{for } |x| < r$



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$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \quad \text{for } |x| < 1$$

Differentiation:
$$\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty} k x^{k-1} \sum_{k=0}^{\infty} (k+1) x^k \text{ for } |x| < 1$$

Integration:
$$-\ln(1-x) = \sum_{k=0}^{\infty} \frac{1}{k+1} x^{k+1} = \sum_{k=1}^{\infty} \frac{1}{k} x^k \text{ for } |x| < 1$$

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ind Integ Examples

Power Series Expansion of ln(1 + x)

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$$\frac{d}{dx} \ln(1+x) = \frac{1}{1+x} = \sum_{k=0}^{\infty} (-1)^k x^k$$
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Integration: $\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} x^{k+1} (+C = 0)$
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The interval of convergence is
$$(-1, 1]$$
. At $x = 1$,

$$\ln 2 = \sum_{k=1}^{\infty} \frac{(-1)^k}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

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$$= \sum_{k=1}^{\infty} \frac{(-1)^k}{k} x^k = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$$

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Power Series Expansion of ln(1 + x)

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Outline

Power Series

- Geometric Series and Variations
- Radius of Convergence

• Differentiation and Integration

- Differentiation and Integration
- Examples

