ODE

Midterm 1 **Math 3331 (Summer 2014)** **June 16, 2014**

Name and ID: _____

50 points 1. Find the solution of the following initial-value problems

a. $\dfrac{dy}{dt} - \dfrac{n}{t}y = e^t t^n, \quad y(1) = 0, \quad (n \text{ integer})$

b. $y'' + 2y' + 2y = 10\cos 2t, \quad y(0) = 0, \quad y'(0) = 1.$

25 points 2. Find the solution of the Logistic equation

$$\frac{dP}{dt} = r\left(1 - \frac{P}{K}\right)P, \quad P(0) = P_0,$$

where r, K and P_0 are constants.

25 points 3. Suppose a population is growing according to the logistic equation

$$\frac{dx}{dt} = f(x) \quad \text{where } f(x) = x - x^2$$

Perform each of the following tasks without the aid of technology.

(i) Sketch a graph of $f(x)$

(ii) Use the graph of f to develop a phase line for the autonomous equation. Classify each equilibrium point as either unstable or asymptotically stable.

(iii) Sketch the equilibrium solutions in the t-x plans. These equilibrium solutions divide the t-x plane into regions. Sketch at least one solution trajectory in each of these regions.

20 points 4. (**BONUS PROBLEM**) An undamped spring-mass system with external driving force is modeled with

$$x'' + 25x = 4\cos 5t.$$

The parameters of this equation are "tuned" so that the frequency of the driving force equals the natural frequency of the undriven system. Suppose that the mass is displaced one positive unit and released from rest.

(a) Find the position of the mass as a function of time. What part of the solution guarantees that this solution resonates?

(b) Sketch the solution found in part (a).

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Problem 1.

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Problem 2.

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Problem 3.

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Problem 4.

When you finish this exam, you should go back and reexamine your work for any errors that you may have made.