ODE

Sample Midterm 2 Math 3331 (Summer 2014) June 24, 2014

20 points 1. Use the Laplace Transform to find the solution of the following initial-value problems

a. $y'' + y = \cos 2t, \quad y(0) = 0, \quad y'(0) = 1.$

b. $y'' - y = e^t, \quad y(0) = 0, \quad y'(0) = 0.$

20 points 2. Consider the initial value problem

$$x' = -x + t, \quad 0 \le t \le 1, \quad x(0) = 0.5. \tag{1}$$

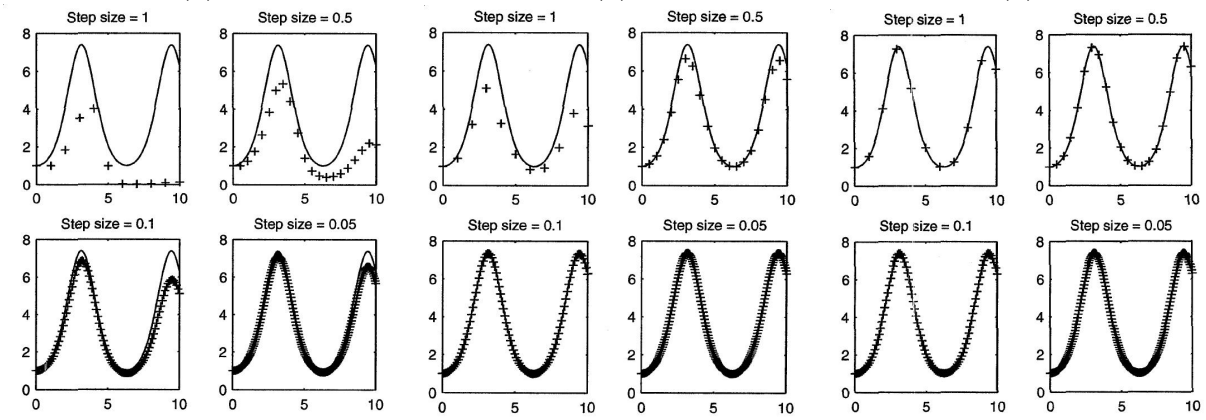
Use the Euler, RK2 and RK4 methods to approximate the value of $x(1)$ for a step size $h = 0.5$ and compute the error of your numerical solution.

20 points 3. Consider the initial value problem

$$x' = x \sin t, \quad t \ge 0, \quad x(0) = 1. \tag{2}$$

The equation is separable and the solution is $x(t) = e^{1-\cos t}$. The Euler method, RK2 and RK4 methods, with step sizes $h = 1$, 0.5, 0.1 and 0.05 produce the following results. Indicate each graph (1,2,3) by its corresponding numerical method and explain your answer.

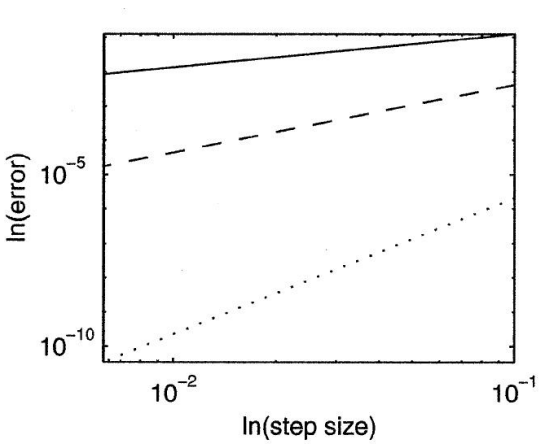
(1) (2) (3)



20 points 4. Consider the initial value problem

$$x' = x, \quad 0 \le t \le 1, \quad x(0) = 1. \tag{3}$$

The equation is separable and the solution is $x(t) = e^t$. We used the Euler method, RK2 and RK4 methods to compute the value of $x(1)$ and constructed a plot of the logarithm of the error versus the logarithm of the step size for each numerical method. The slope of the solid line is 0.9716, the slope of the dashed line is 1.9755, and the slope of the dotted line is 3.9730. Indicate each line by its corresponding numerical method and explain your answer.



20 points 5. Write each initial value problems as a system of the first-order equations using vector notation.

a. $x'' + \delta x' - x + x^3 = \gamma \cos \omega t, \quad x(0) = x_0,\ x'(0) = v_0$

b. $x'' + \mu(x^2 - 1)x' + x = 0, \quad x(0) = x_0,\ x'(0) = v_0$