Math 3331 Differential Equations

2.5 Mixing Problems

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2..5 Mixing Problems

- Balance Law
- Mixture of Water and Salt
  - Example 5.1
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Mixing Problems

Solution of a mixture of water and salt

$x(t)$: amount of salt

$V(t)$: volume of the solution

$c(t)$: concentration of salt

$$\Rightarrow \quad c(t) = \frac{x(t)}{V(t)}$$

Balance Law

$$\frac{dx}{dt} = \text{rate in} - \text{rate out}$$

$$\text{rate} = \text{flow rate} \times \text{concentration}$$

Example 1

See Text, Example 2.5.1

$V(t) = 100$ gal, kept constant

concentration in = 2 lb/gal

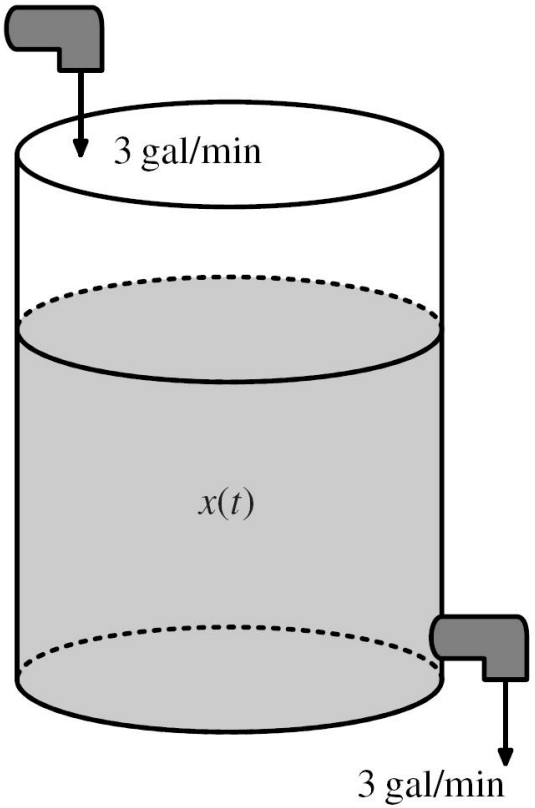
flow rate in = 3 gal/min

flow rate out = flow rate in

$\Rightarrow \quad c(t) = x(t)/100$ lb/gal

$\Rightarrow \quad$ rate in $= 6$ lb/min

$\Rightarrow \quad$ rate out $= 3x(t)/100$ lb/min



Balance Law

$$\frac{dx}{dt} = 6 - 3x/100$$

Example 2

See Text, Example 2.5.2

concentration in = 1.5 lb/gal

flow rate in = 3 gal/min

flow rate out = 1 gal/min

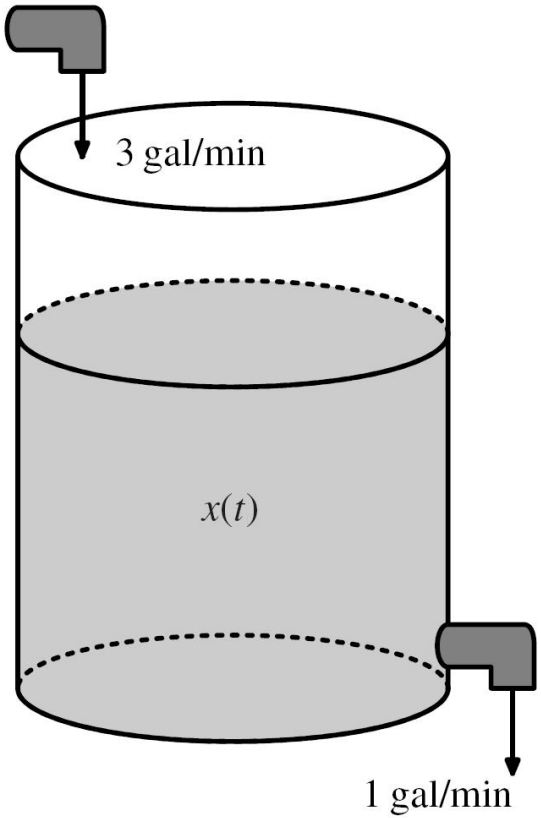
$V(0) = 300$ gal

$\Rightarrow \quad V(t) = 300 + 2t$ lb/gal

$\Rightarrow \quad c(t) = x(t)/(300 + 2t)$ lb/gal

$\Rightarrow \quad$ rate in $= 4.5$ lb/min

$\Rightarrow \quad$ rate out $= x(t)/(300 + 2t)$ lb/min



Balance Law

$$\frac{dx}{dt} = 4.5 - x/(300 + 2t)$$