

HW13 Solutions

Math 3338-10853: Probability (Fall 2006), Dr. Jiwen He

18.

$$\begin{aligned} \text{a. } E(X + Y) &= \sum_x \sum_y (x + y)p(x, y) = (0 + 0)(.02) \\ &+ (0 + 5)(.06) + \dots + (10 + 15)(.01) = 14.10 \end{aligned}$$

$$\begin{aligned} \text{b. } E[\max(X, Y)] &= \sum_x \sum_y \max(x, y) \cdot p(x, y) \\ &= (0)(.02) + (5)(.06) + \dots + (15)(.01) = 9.60 \end{aligned}$$

22.

$$\begin{aligned} \text{Revenue} &= 3X + 10Y, \text{ so } E(\text{revenue}) = E(3X + 10Y) \\ &= \sum_{x=0}^5 \sum_{y=0}^2 (3x + 10y) \cdot p(x, y) = 0 \cdot p(0, 0) + \dots + 35 \cdot p(5, 2) = 15.4 \end{aligned}$$

$$\begin{aligned} \text{25. } \text{Cov}(X, Y) &= -\frac{2}{75} \text{ and } \mu_x = \mu_y = \frac{2}{5}. \quad E(X^2) = \int_0^1 x^2 \cdot f_x(x) dx \\ &= 12 \int_0^1 x^3 (1 - x^2) dx = \frac{12}{60} = \frac{1}{5}, \text{ so } \text{Var}(X) = \frac{1}{5} - \frac{4}{25} = \frac{1}{25} \\ \text{Similarly, } \text{Var}(Y) &= \frac{1}{25}, \text{ so } \rho_{X, Y} = \frac{-2/75}{\sqrt{1/25} \cdot \sqrt{1/25}} = -\frac{50}{75} = -.667 \end{aligned}$$

29.

$$\begin{aligned} \text{Since } E(XY) &= E(X) \cdot E(Y), \text{ Cov}(X, Y) = E(XY) - E(X) \cdot E(Y) = E(X) \cdot E(Y) - E(X) \cdot E(Y) = \\ 0, \text{ and since } \text{Corr}(X, Y) &= \frac{\text{Cov}(X, Y)}{\sigma_x \sigma_y}, \text{ then } \text{Corr}(X, Y) = 0 \end{aligned}$$