# HW15 Solutions

## Math 3338-10853: Probability (Fall 2006), Dr. Jiwen He

**28.**

**a.** $E(X_1 + X_2 + X_3) = 180$, $V(X_1 + X_2 + X_3) = 45$, $\sigma_{x_1+x_2+x_3} = 6.708$

$P(X_1 + X_2 + X_3 \le 200) = P\left(Z \le \frac{200-180}{6.708}\right) = P(Z \le 2.98) = .9986$

$P(150 \le X_1 + X_2 + X_3 \le 200) = P(-4.47 \le Z \le 2.98) \approx .9986$

**b.** $\mu_{\overline{X}} = \mu = 60$, $\sigma_{\overline{X}} = \frac{\sigma_x}{\sqrt{n}} = \frac{\sqrt{15}}{\sqrt{3}} = 2.236$

$P(\overline{X} \ge 55) = P\left(Z \ge \frac{55-60}{2.236}\right) = P(Z \ge -2.236) = .9875$

$P(58 \le \overline{X} \le 62) = P(-.89 \le Z \le .89) = .6266$

**c.** $E(X_1 - .5X_2 - .5X_3) = 0$;

$V(X_1 - .5X_2 - .5X_3) = \sigma_1^2 + .25\sigma_2^2 + .25\sigma_3^2 = 22.5$, sd = 4.7434

$P(-10 \le X_1 - .5X_2 - .5X_3 \le 5) = P\left(\frac{-10-0}{4.7434} \le Z \le \frac{5-0}{4.7434}\right)$

$= P(-2.11 \le Z \le 1.05) = .8531 - .0174 = .8357$

**d.** $E(X_1 + X_2 + X_3) = 150$, $V(X_1 + X_2 + X_3) = 36$, $\sigma_{x_1+x_2+x_3} = 6$

$P(X_1 + X_2 + X_3 \le 200) = P\left(Z \le \frac{160-150}{6}\right) = P(Z \le 1.67) = .9525$

We want $P(X_1 + X_2 \ge 2X_3)$, or written another way, $P(X_1 + X_2 - 2X_3 \ge 0)$

$E(X_1 + X_2 - 2X_3) = 40 + 50 - 2(60) = -30$,

$V(X_1 + X_2 - 2X_3) = \sigma_1^2 + \sigma_2^2 + 4\sigma_3^2 = 78$, sd = 8.832, so

$P(X_1 + X_2 - 2X_3 \ge 0) = P\left(Z \ge \frac{0-(-30)}{8.832}\right) = P(Z \ge 3.40) = .0003$

**29.** Y is normally distributed with $\mu_Y = \frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{3}(\mu_3 + \mu_4 + \mu_5) = -1$, and

$\sigma_Y^2 = \frac{1}{4}\sigma_1^2 + \frac{1}{4}\sigma_2^2 + \frac{1}{9}\sigma_3^2 + \frac{1}{9}\sigma_4^2 + \frac{1}{9}\sigma_5^2 = 3.167, \sigma_Y = 1.7795$.

Thus, $P(0 \le Y) = P\left(\frac{0-(-1)}{1.7795} \le Z\right) = P(.56 \le Z) = .2877$ and

$P(-1 \le Y \le 1) = P\left(0 \le Z \le \frac{2}{1.7795}\right) = P(0 \le Z \le 1.12) = .3686$

**33.** Let $X_1, \ldots, X_5$ denote morning times and $X_6, \ldots, X_{10}$ denote evening times.

**a.** $E(X_1 + \ldots + X_{10}) = E(X_1) + \ldots + E(X_{10}) = 5\,E(X_1) + 5\,E(X_6)$
$= 5(4) + 5(5) = 45$

**b.** $Var(X_1 + \ldots + X_{10}) = Var(X_1) + \ldots + Var(X_{10}) = 5\,Var(X_1) + 5Var(X_6)$

$$= 5\left[\frac{64}{12} + \frac{100}{12}\right] = \frac{820}{12} = 68.33$$

**c.** $E(X_1 - X_6) = E(X_1) - E(X_6) = 4 - 5 = -1$

$Var(X_1 - X_6) = Var(X_1) + Var(X_6) = \dfrac{64}{12} + \dfrac{100}{12} = \dfrac{164}{12} = 13.67$

**d.** $E[(X_1 + \ldots + X_5) - (X_6 + \ldots + X_{10})] = 5(4) - 5(5) = -5$
$Var[(X_1 + \ldots + X_5) - (X_6 + \ldots + X_{10})]$
$= Var(X_1 + \ldots + X_5) + Var(X_6 + \ldots + X_{10}) = 68.33$

**34.** $\mu = 5.00$, $\sigma = .2$

**a.** $E(\overline{X} - \overline{Y}) = 0;$ $\quad V(\overline{X} - \overline{Y}) = \dfrac{\sigma^2}{25} + \dfrac{\sigma^2}{25} = .0032$, $\sigma_{\overline{X}-\overline{Y}} = .0566$

$$\Rightarrow P(-.1 \le \overline{X} - \overline{Y} \le .1) = P(-1.77 \le Z \le 1.77) = .9232$$

**b.** $V(\overline{X} - \overline{Y}) = \dfrac{\sigma^2}{36} + \dfrac{\sigma^2}{36} = .0022222$, $\sigma_{\overline{X}-\overline{Y}} = .0471$

$\Rightarrow P(-.1 \le \overline{X} - \overline{Y} \le .1) \approx P(-2.12 \le Z \le 2.12) = .9660$ (by the CLT)

**37.** Let $X_1$ and $X_2$ denote the (constant) speeds of the two planes.

**a.** After two hours, the planes have traveled $2X_1$ km. and $2X_2$ km., respectively, so the second will not have caught the first if $2X_1 + 10 > 2X_2$, i.e. if $X_2 - X_1 < 5$. $X_2 - X_1$ has a mean $500 - 520 = -20$, variance $100 + 100 = 200$, and standard deviation 14.14. Thus,

$$P(X_2 - X_1 < 5) = P\left(Z < \frac{5-(-20)}{14.14}\right) = P(Z < 1.77) = .9616.$$

**b.** After two hours, #1 will be $10 + 2X_1$ km from where #2 started, whereas #2 will be $2X_2$ from where it started. Thus the separation distance will be al most 10 if $|2X_2 - 10 - 2X_1| \le 10$, i.e. $-10 \le 2X_2 - 10 - 2X_1 \le 10$,
i.e. $0 \le X_2 - X_1 \le 10$. The corresponding probability is
$P(0 \le X_2 - X_1 \le 10) = P(1.41 \le Z \le 2.12) = .9830 - .9207 = .0623.$

**39.**

**a.** $E(Y_i) = .5$, so $E(W) = \sum_{i=1}^{n} i \cdot E(Y_i) = .5\sum_{i=1}^{n} i = \dfrac{n(n+1)}{4}$

**b.** $Var(Y_i) = .25$, so $Var(W) = \sum_{i=1}^{n} i^2 \cdot Var(Y_i) = .25\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{24}$