HW3 Solutions
Math 3338-10853: Probability (Fall 2006), Dr. Jiwen He

31.

a. (5)(4) = 20 (5 choices for president, 4 remain for vice president)

b. (5)(4)(3) = 60

c. $\binom{5}{2} = \frac{5!}{2!3!} = 10$ (No ordering is implied in the choice)

36.

a. $\binom{20}{6} = 38{,}760.$ P(all from day shift) $= \frac{\binom{20}{6}\binom{25}{0}}{\binom{45}{6}} = \frac{38{,}760}{8{,}145{,}060} = .0048$

b. P(all from same shift) $= \frac{\binom{20}{6}\binom{25}{0}}{\binom{45}{6}} + \frac{\binom{15}{6}\binom{30}{0}}{\binom{45}{6}} + \frac{\binom{10}{6}\binom{35}{0}}{\binom{45}{6}}$

= .0048 + .0006 + .0000 = .0054

c. P(at least two shifts represented) = 1 – P(all from same shift)
= 1 - .0054 = .9946

d. Let A_1 = day shift unrepresented, A_2 = swing shift unrepresented, and A_3 = graveyard shift unrepresented. Then we wish $P(A_1 \cup A_2 \cup A_3)$.
$P(A_1)$ = P(day unrepresented) = P(all from swing and graveyard)

$P(A_1) = \frac{\binom{25}{6}}{\binom{45}{6}}$, $P(A_2) = \frac{\binom{30}{6}}{\binom{45}{6}}$, $P(A_3) = \frac{\binom{35}{6}}{\binom{45}{6}}$,

$P(A_1 \cap A_2) = P(\text{all from graveyard}) = \dfrac{\binom{10}{6}}{\binom{45}{6}}$

$P(A_1 \cap A_3) = \dfrac{\binom{15}{6}}{\binom{45}{6}}$, $P(A_2 \cap A_3) = \dfrac{\binom{20}{6}}{\binom{45}{6}}$, $P(A_1 \cap A_2 \cap A_3) = 0$,

So $P(A_1 \cup A_2 \cup A_3) = \dfrac{\binom{25}{6}}{\binom{45}{6}} + \dfrac{\binom{30}{6}}{\binom{45}{6}} + \dfrac{\binom{35}{6}}{\binom{45}{6}} - \dfrac{\binom{10}{6}}{\binom{45}{6}} - \dfrac{\binom{15}{6}}{\binom{45}{6}} - \dfrac{\binom{20}{6}}{\binom{45}{6}}$

$= .2939 - .0054 = .2885$

45.

a. P(A) = .106 + .141 + .200 = .447, P(C) =.215 + .200 + .065 + .020 = .500 $P(A \cap C)$ = .200

b. $P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{.200}{.500} = .400$. If we know that the individual came from ethnic group 3, the probability that he has type A blood is .40. $P(C|A) = \dfrac{P(A \cap C)}{P(A)} = \dfrac{.200}{.447} = .447$. If a person has type A blood, the probability that he is from ethnic group 3 is .447

c. Define event D = {ethnic group 1 selected}. We are asked for $P(D|B') = \dfrac{P(D \cap B')}{P(B')} = \dfrac{.192}{.909} = .211$. $P(D \cap B')$=.082 + .106 + .004 = .192, $P(B') = 1 - P(B) = 1 - [.008 + .018 + .065] = .909$

48.

a. $P(A_2 | A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{.06}{.12} = .50$

b. $P(A_1 \cap A_2 \cap A_3 | A_1) = \dfrac{.01}{.12} = .0833$

c. We want P[(exactly one) | (at least one)].

P(at least one) $= P(A_1 \cup A_2 \cup A_3)$

$= .12 + .07 + .05 - .06 - .03 - .02 + .01 = .14$

Also notice that the intersection of the two events is just the 1st event, since "exactly one" is totally contained in "at least one."

So P[(exactly one) | (at least one)]= $\dfrac{.04+.01}{.14} = .3571$

d. The pieces of this equation can be found in your answers to exercise 26 (section 2.2):

$$P(A_3' \mid A_1 \cap A_2) = \frac{P(A_1 \cap A_2 \cap A_3')}{P(A_1 \cap A_2)} = \frac{.05}{.06} = .833$$

54. $P(A_1) = .22$, $P(A_2) = .25$, $P(A_3) = .28$, $P(A_1 \cap A_2) = .11$, $P(A_1 \cap A_3) = .05$, $P(A_2 \cap A_3) = .07$, $P(A_1 \cap A_2 \cap A_3) = .01$

a. $P(A_2 | A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{.11}{.22} = .50$

b. $P(A_2 \cap A_3 | A_1) = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \dfrac{.01}{.22} = .0455$

c. $$P(A_2 \cup A_3 \mid A_1) = \frac{P[A_1 \cap (A_2 \cup A_3)]}{P(A_1)} = \frac{P[(A_1 \cap A_2) \cup (A_1 \cap A_3)]}{P(A_1)}$$

$$= \frac{P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{.15}{.22} = .682$$

d. $$P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \frac{.01}{.53} = .0189$$

This is the probability of being awarded all three projects given that at least one project was awarded.