HW7 Solutions

Math 3338-10853: Probability (Fall 2006), Dr. Jiwen He

58.

a. $b(3;8,.6) = \binom{8}{3}(.6)^3(.4)^5 = (56)(.00221184) = .124$

b. $b(5;8,.6) = \binom{8}{5}(.6)^5(.4)^3 = (56)(.00497664) = .279$

c. $P(3 \le X \le 5) = b(3;8,.6) + b(4;8,.6) + b(5;8,.6) = .635$

d. $P(1 \le X) = 1 - P(X = 0) = 1 - \binom{12}{0}(.1)^0(.9)^{12} = 1 - (.9)^{12} = .718$

60. $X \sim Bin(25, .05)$

a. $P(X \le 2) = B(2;25,.05) = .873$

b. $P(X \ge 5) = 1 - P(X \le 4) = 1 - B(4;25,.05) = .1 - .993 = .007$

c. $P(1 \le X \le 4) = P(X \le 4) - P(X \le 0) = .993 - .277 = .716$

d. $P(X = 0) = P(X \le 0) = .277$

e. $E(X) = np = (25)(.05) = 1.25$
$V(X) = np(1 - p) = (25)(.05)(.95) = 1.1875$
$\sigma_x = 1.0897$

66. $X \sim Bin(25, .02)$

a. $P(X=1) = 25(.02)(.98)^{24} = .308$

b. $P(X \ge 1) = 1 - P(X=0) = 1 - (.98)^{25} = 1 - .603 = .397$

c. $P(X \ge 2) = 1 - P(X \le 1) = 1 - [.603 + .308] = .089$

d. $\mu = 25(.02) = .5$; $\sigma = \sqrt{npq} = \sqrt{25(.02)(.98)} = \sqrt{.49} = .7$
$\mu + 2\sigma = .5 + 1.4 = 1.9$ So $P(0 \le X \le 1.9 = P(X \le 1) = .911$

e. $\dfrac{.5(4.5) + 24.5(3)}{25} = 3.03$ hours

69.

a. P(rejecting claim when p = .8) = B(15;25,.8) = .017

b. P(not rejecting claim when p = .7) = P(X $\geq$ 16 when p = .7)
= 1 - B(15;25,.7) = 1 - .189 = .811; for p = .6, this probability is
= 1 - B(15;25,.6) = 1 - .575 = .425.

c. The probability of rejecting the claim when p = .8 becomes B(14;25,.8) = .006, smaller than in **a** above. However, the probabilities of **b** above increase to .902 and .586, respectively.

77. When p = .5, μ = 10 and σ = 2.236, so 2σ = 4.472 and 3σ = 6.708.
The inequality |X – 10| $\geq$ 4.472 is satisfied if either X $\leq$ 5 or X $\geq$ 15, or P(|X - μ| $\geq$ 2σ) = P(X $\leq$ 5 or X $\geq$ 15) = .021 + .021 = .042. The inequality |X – 10| $\geq$ 6.708 is satisfied if either X $\leq$ 3 or X $\geq$ 17, so P(|X – μ| $\geq$ 3σ) = P(X $\leq$3 or X $\geq$ 17) = .001 + .001 = .002.

In the case p = .75, μ = 15 and σ = 1.937, so 2σ = 3.874 and 3σ = 5.811. P(|X - 15| $\geq$ 3.874) = P(X $\leq$ 11 or X $\geq$ 19) = .041 + .024 = .065, whereas P(|X - 15| $\geq$ 5.811) = P(X $\leq$ 9) = .004.

All these probabilities are considerably less than the upper bounds .25 (for k = 2) and .11 (for k = 3) given by Chebyshev.

80.

a. X ~ Hypergeometric N=15, n=5, M=6

b. $P(X=2) = \dfrac{\binom{6}{2}\binom{9}{3}}{\binom{15}{5}} = \dfrac{1260}{3003} = .420$

$P(X\leq 2) = P(X=0) + P(X=1) + P(X=2)$

$$= \frac{\binom{9}{5}}{\binom{15}{5}} + \frac{\binom{6}{1}\binom{9}{4}}{\binom{15}{5}} + \frac{1260}{3003} = \frac{126+756+1260}{3003} = \frac{2142}{3003} = .713$$

$P(X\geq 2) = 1 - P(X\leq 1) = 1 - [P(X=0) + P(X=1)] = 1 - \dfrac{126+756}{3003} = .706$

c. $E(X) = 5\left(\dfrac{6}{15}\right) = 2$; $V(X) = \left(\dfrac{15-5}{14}\right)\cdot 5\cdot\left(\dfrac{6}{15}\right)\cdot\left(1-\dfrac{6}{15}\right) = .857$;

$\sigma = \sqrt{V(X)} = .926$

83.

a. Possible values of X are 5, 6, 7, 8, 9, 10. (In order to have less than 5 of the granite, there would have to be more than 10 of the basaltic).

$$P(X = 5) = h(5;\ 15,10,20) = \frac{\binom{10}{5}\binom{10}{10}}{\binom{20}{15}} = .0163 .$$

Following the same pattern for the other values, we arrive at the pmf, in table form below.

x	5	6	7	8	9	10
p(x)	.0163	.1354	.3483	.3483	.1354	.0163

b. P(all 10 of one kind or the other) = P(X = 5) + P(X = 10) = .0163 + .0163 = .0326

c. $E(X) = n \cdot \frac{M}{N} = 15 \cdot \frac{10}{20} = 7.5$; $V(X) = \left(\frac{5}{19}\right)(7.5)\left(1 - \frac{10}{20}\right) = .9868$;

$\sigma_x = .9934$

$\mu \pm \sigma = 7.5 \pm .9934 = (6.5066, 8.4934)$, so we want
P(X = 7) + P(X = 8) = .3483 + .3483 = .6966

91. $M_X(t) = \frac{p^r}{(1-qe^t)^r}$, so $M'_X(0) = \left.\frac{rqp^r e^t}{(1-qe^t)^{r+1}}\right|_{t=0} = \frac{rqp^r}{(1-q)^{r+1}} = \frac{rqp^r}{p^{r+1}} = \frac{rq}{p}$. Next, $E(X^2) =$

$M''_X(0) = \left.\frac{rqp^r[rqe^{2t} + e^t]}{(1-qe^t)^{r+2}}\right|_{t=0} = \cdots = \frac{rq(rq+1)}{p^2}$, from which $V(X) =$

$\frac{rq(rq+1)}{p^2} - \left[\frac{rq}{p}\right]^2 = \frac{rq}{p^2}$.