General Remarks

- Staple your work together.
- Write neatly.

Section 1.2 Problem Notes

(1) (Problem 1.2.13) When you use the result that $0 \cdot v = \vec{0}$, you need to either prove the result or state that it is from Theorem 1.2 (a). This is a general requirement for proofs in this class: every claim either has to be proven or you must appeal to a result from class or the textbook.

(2) (Problem 1.2.13) You have to be careful about the zero vector for this problem. Since $(a, b)+(0, 0) = (a+0, b\cdot 0) = (a, 0) \neq (a, b)$ then we have that $(0, 0)$ is NOT the zero vector in this vector space. Instead, $(0, 1)$ is since $(a, b) + (0, 1) = (a + 0, b \cdot 1) = (a, b)$. Thus you must be cautious about (VS3); just because there is not an obvious "zero" vector doesn't mean there isn't one, this would be something you have to prove.

(3) (Problem 1.2.13) This problem shows that the set V can have different addition and scalar multiplication operations, which may give rise to different vector space structures for V. So, just because $(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2)$ in the usual vector space structure for V doesn't tell us that this problem's addition and scalar multiplication operations are wrong. Because the operations are different, we get different facts, like a zero vector of $(0, 1)$ and failure of (VS4) in part because $(-a, -b)$ is no longer an additive inverse of (a, b) with these operations (and furthermore there are elements without any additive inverse).

(4) (Problem 1.2.16) Luckily for some of you, this problem wasn't graded. You cannot just answer a question like this with a simple yes or no. You have to explain your reasoning.

Section 1.3 Problem Notes

(5) (1.3.19) *A reminder about the structure of an iff. proof.* Statements of the form "statement A iff. statement B" have two directions of proof. For example, in this question you **first** ($\Longrightarrow$) assume that $W_1 \cup W_2$ is a vector space and then you show that this fact implies that one of the vector spaces had to have been contained in the other. **Second** ($\Longleftarrow$, which here is the easy direction), you assume that one of the subspaces is actually contained in the other and use this assumption to show that their union is a vector space.

- The most common way that people showed the first direction for this problem, is by contradiction: assume $W_1 \cup W_2$ is a vector space but neither subspace is contained in the other. This means we can find $x \in W_1 \setminus W_2$ and $y \in W_2 \setminus W_1$, i.e., x and y are only in one of the subspaces. Since $W_1 \cup W_2$ is a vector space then $x + y \in W_1 \cup W_2$ so either $x + y \in W_1$ or $x + y \in W_2$. In the first case, $(x + y) - x = y \in W_1$. But this contradicts our choice of y, namely we chose $y \notin W_1$. Hence our assumption that neither subspace is contained in the other must be false, showing this direction. [1]
- An alternative way to showing the first direction directly is showing the contrapositive: $\neg B \implies \neg A$. In this case this would be first assuming that neither of $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$ is true and then using this assumption to show that $W_1 \cup W_2$ is not a subspace.
- When I write "iff." and take off points, it usually means you forgot to show one of the directions.

[1] A common error was made where you chose $x \in W_1$ and $y \in W_2$. Then as before $x + y \in W_1$ OR $x + y \in W_2$, which implies either $y \in W_1$ or $x \in W_2$. This fails to show containment because maybe some choice of x always forces $y \in W_1$ but not $x \in W_2$ and some special choice of y does the opposite, breaking either possibility of containment.