Math 6371-13999 (Spring 2011): Numerical Analysis *

Instructor:	Dr. Jiwen He
Office:	684 PGH
Phone:	(713) 743-3481
e-mail:	jiwenhe@math.uh.edu
Time:	1:00PM-2:30PM TuTh
Room:	202 AH
Office hours:	1:00PM-2:00PM MW or by appointment

Texts

- *Numerical Methods in Scientific Computing*, Volume 1, Germund Dahlquist and Ake Bjorck, SIAM, 2008, ISBN: 978-0-898716-44-3.
- *A First Course in the Numerical Analysis of Differential Equations*, 2nd Edition, Arieh Iserles, Cambridge University Press, 2008, ISBN 978-0-521734-90-5.

Brief Description

This is the second semester of a two semester course. The focus in this semester is on approximation theory and numerical analysis of both ordinary and partial differential equations. The applications of approximation theory to interpolation, Fourier analysis, numerical differentiation and Gaussian integration will be addressed. The concepts of consistency, convergence, stability for the numerical solution of ODEs will be discussed. Other topics covered include multistep and Runge-Kutta methods; finite difference and finite elements techniques for the Poisson equation; and a variety of algorithms to solve large, sparse algebraic systems.

*This syllabus contains important information about this course to which you will need to refer from time to time.

Course Outline, Homework, and Exam Dates[1]

- *Numerical Methods in Scientific Computing*, Volume 1, Germund Dahlquist and Ake Bjorck, SIAM, 2008, ISBN: 978-0-898716-44-3.

- ## 1 Principles of Numerical Calculations
  - 1.1 Common Ideas and Concepts
    * 1.1.1 Fixed-Point Iteration, 1.1.2 Newtons Method, 1.1.3 Linearization and Extrapolation, 1.1.4 Finite Difference Approximations
  - 1.5 Numerical Solution of Differential Equations
    * 1.5.1 Eulers Method, 1.5.2 An Introductory Example, 1.5.3 Second Order Accurate Methods

- ## 4 Interpolation and Approximation
  - 4.1 The Interpolation Problem
    * 4.1.1 Introduction, 4.1.2 Bases for Polynomial Interpolation, 4.1.3 Conditioning of Polynomial Interpolation
  - 4.2 Interpolation Formulas and Algorithms
    * 4.2.1 Newtons Interpolation Formula, 4.2.3 Barycentric Lagrange Interpolation, 4.2.4 Iterative Linear Interpolation, 4.2.5 Fast Algorithms for Vandermonde Systems, 4.2.6 The Runge Phenomenon
  - 4.3 Generalizations and Applications
    * 4.3.1 Hermite Interpolation, 4.3.4 Multivariate Interpolation

- Review Questions I: 1.1.5, 1.2.3, 1.3.4, 1.4.3, 1.5.2, 1.6.1, 2.1.2, 2.2.5, 2.3.3, 2.4.2, 2.4.6, 4.1.6, 4.2.4, 4.3.1

- Problems and Computer Exercises I: 1.1.3, 1.1.5, 1.5.3, 4.1.1, 4.2.3, 4.3.1, 4.3.3
  - 4.4 Piecewise Polynomial Interpolation
    * 4.4.1 Berntein Polynomials and Bzier Curves, 4.4.2 Spline Functions
  - 4.5 Approximation and Function Spaces
    * 4.5.3 Inner Product Spaces and Orthogonal Systems, 4.5.4 Solution of the Approximation Problem, 4.5.5 Mathematical Properties of Orthogonal Polynomials

[1] The dates of exams are subject to change. Changes will be announced in class.

  - 4.6 Fourier Methods
    * 4.6.1 Basic Formulas and Theorems, 4.6.2 Discrete Fourier Analysis,
  - 4.7 The Fast Fourier Transform
    * 4.7.1 The Fast Fourier Algorithm, 4.7.2 Discrete Convolution by FFT

- Review Questions II: 3.1.1, 3.2.3, 3.3.4, 3.4.5, 4.4.1, 4.5.7, 4.6.1, 4.7.1

- Problems and Computer Exercises II: 4.4.2, 4.4.4, 4.5.6, 4.5.18, 4.6.10, 4.7.2

- **5 Numerical Integration**
  - 5.1 Interpolatory Quadrature Rules
    * 5.1.1 Introduction, 5.1.2 Treating Singularities, 5.1.3 Some Classical Formulas, 5.1.4 Superconvergence of the Trapezoidal Rule
  - 5.2 Integration by Extrapolation
    * 5.2.1 The EulerMaclaurin Formula, 5.2.2 Rombergs Method, 5.2.3 Oscillating Integrands
  - 3.4 Acceleration of Convergence
    * 3.4.1 Introduction, 3.4.2 Comparison Series and Aitken Acceleration, 3.4.3 Eulers Transformation, 3.4.5 EulerMaclaurins Formula, 3.4.6 Repeated Richardson Extrapolation
  - 5.3 Quadrature Rules with Free Nodes
    * 5.3.1 Method of Undetermined Coefficients, 5.3.2 GaussChristoffel Quadrature Rules, 5.3.4 Matrices, Moments, and Gauss Quadrature, 5.3.5 Jacobi Matrices and Gauss Quadrature
  - Lecture 34. From Lanczos to Gauss Quadrature, *Numerical Linear Algebra*, Lloyd N . Trefethen and David Bau, SIAM, 1997, ISBN: 0898713619.

- Review Questions III: 5.1.3, 5.1.5, 5.2.2, 5.2.4, 3.4.5, 5.3.2, 5.3.4

- Problems and Computer Exercises III: 5.1.10, 5.1.16, 5.2.2, 5.2.7, 3.4.8, 5.3.3, 5.3.11

- **EXAM I (March 24)**

- *A First Course in the Numerical Analysis of Differential Equations*, 2nd Edition, Arieh Iserles, Cambridge University Press, 2008, ISBN 978-0-521734-90-5.

- **Part I Ordinary differential equations**
  - 1. Euler's method and beyond
    - 1.1 Ordinary differential equations and the Lipschitz condition
    - 1.2 Eulers method
    - 1.3 The trapezoidal rule
    - 1.4 The theta method
  - 2. Multistep methods
    - 2.1 The Adams method
    - 2.2 Order and convergence of multistep methods
    - 2.3 Backward differentiation formulae
  - 3. RungeKutta methods
    - 3.2 Explicit RungeKutta schemes
    - 3.3 Implicit RungeKutta schemes
  - 4. Stiff equations
    - 4.1 What are stiff ODEs?
    - 4.2 The linear stability domain and A-stability
    - 4.3 A-stability of RungeKutta methods
    - 4.3 A-stability of RungeKutta methods
- **PartII The Poisson equation**
  - 8. Finite difference schemes
    - 8.1 Finite differences
    - 8.2 The five-point formula for $\nabla^2 u = f$
  - 9. The finite element method
    - 9.1 Two-point boundary value problems
    - 9.2 A synopsis of FEM theory
    - 9.3 The Poisson equation
  - 12. Classical iterative methods for sparse linear equations
    - 12.1 Linear one-step stationary schemes
    - 12.2 Classical iterative methods
- Problems and Computer Exercises IV: 1.2, 2.3, 3.4, 4.2
- **EXAM II (May 5)**