

Math 1431 Spring 2006

Review for Exam 1

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1. Compute the following limits. Justify your answers. (The limits may not exist)

(a)  $\lim_{x \rightarrow 1} [(x - 1)^2 \sin \frac{1}{x-1}]$

(b)  $\lim_{x \rightarrow 2} \frac{\sin 4(x-2)}{x-2}$

(c)  $\lim_{x \rightarrow 0} \frac{\tan x}{\sin x}$

(d)  $\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2}$

(e)  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x}{x - \frac{\pi}{2}}$

2. Find an interval of length 1 that contains a solution of the equation  $-x^3 + x + 7 = 0$ . What property of the function  $f(x) = -x^3 + x + 7$  and which theorem(s) are you using? (Note: I'm not asking you to find the solution).

3. Find the equation of the line tangent to the curve  $\frac{x^2}{4} + \frac{y^2}{16} = 1$  at the point  $(\sqrt{2}, -2\sqrt{2})$ .

4. A car is driving east away from a traffic light at the rate of 30 feet per second. A police is driving north towards the same light at a rate of 20 feet per second. How fast is the distance between the two cars changing when the police car is 300 feet from the light and the other car is 400 feet from the light?

5. In each of the following, find  $f'(x)$ .

(a)  $f(x) = \frac{x^{4/3} \tan x^2}{x^2 + 1}$

(b)  $f(x) = \sqrt{\sec x^2}$

6. Suppose that  $y$  is a function of  $x$  that satisfies  $x^2 \sin y + x^3 y^2 = 1 - x$ . Find  $\frac{dy}{dx}$ .

7. Let  $f$  be defined as

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 4 \\ ax - b & \text{if } x > 4 \end{cases} .$$

Find  $a$  and  $b$  such that  $f$  is **differentiable** at  $x = 4$ .

8. A light source is on top of a 75 foot pole. A ball is falling vertically so that it will strike the ground 20 feet from the pole. As the ball falls, its shadow moves along the ground. If the ball is falling at 30 feet per second when it is 45 feet above the ground, how fast and in which direction is the shadow moving at that time?

9. Let  $f(x) = 5x - \frac{1}{x} + 1$ . Find the slope of this function at  $x = 1$ .

10. Let  $f(x) = \sin 5x$ . Compute the first 5 derivatives of  $f$ .

11. Suppose that the height  $h$  of a bouncing ball at time  $t$  is as given below.

$$h(t) = \begin{cases} -16t^2 + 64 & \text{if } 0 \leq t \leq 2 \\ -16t^2 + 96t - 128 & \text{if } 2 \leq t \leq 4 \\ -16t^2 + 144t - 320 & \text{if } 4 \leq t \leq 5 \end{cases}$$

(a) Find the velocity  $v(t) = h'(t)$  of the bouncing ball in the form

$$h(t) = \begin{cases} ? & \text{if } 0 < t < 2 \\ ? & \text{if } 2 < t < 4 \\ ? & \text{if } 4 < t < 5 \end{cases}$$

(b) Compute  $\lim_{t \rightarrow 4^-} v(t)$  and  $\lim_{t \rightarrow 4^+} v(t)$ .

(c) Is the velocity function  $v$  continuous at  $t = 4$ ? Explain.

(d) Is the height function  $h$  continuous at  $t = 4$ ? Explain why or why not.