June 21, 2008

Name:

Midterm Math 5336

1. Express in the form $P \to Q$ or $Q \to P$ the following statements:

   a) P in case that Q.
   b) P only if Q
   c) P if Q.
   d) P is necessary for Q.
   e) P is sufficient for Q.
   f) P whenever Q

2. State whether the formula is a tautology or not.

   (a) $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

   (b) $((p \wedge \neg p) \to q)$

   (c) $(p \to (q \to r)) \to ((p \to q) \to r))$

   (d) $((p \vee q) \wedge \neg p) \to q$

3. Determine whether the following arguments are valid or invalid.

   (a) If it is raining, then I will stay home. It is not raining. Thus I will not stay home.

   (b) If it is raining, then I will stay home. I am not staying home. Thus it is not raining

   (c) I will stay home only if it is raining. It is not raining. Thus I will not stay home.

   (d) Only citizens can vote. Paul cannot vote. So he is not a citizen.

4. True or false:

   (a) A function f from the set A to the set B is injective (or one-tone) if $f(x) \neq f(y)$ implies $x \neq y$.

   (b) A function f from the set A to the set B is injective (or one-tone) if $f(x) \neq f(y)$ in case that $x \neq y$.

5. Find a function $f : \mathbb{N} \to \mathbb{N}$ which is injective but not surjective and a function $g : \mathbb{N} \to \mathbb{N}$ which is surjective but not injective.

6. Let A be a **finite** set. Say A has five elements.

   (a) Can you find a function $g : A \to A$ which is injective but not surjective? Explain your answer.

   (b) Can you find a function $f : A \to A$ which is surjective but not injective? Explain your answer.

7. Are there any subsets of the empty set $\emptyset$?

8. Let B be a set such that for every set A you have that $A \cup B = A$. What can you say about B?

9. What is the number of elements of the set $\{\emptyset, \{\{\emptyset\}\}\}$?

10. State the Method of Mathematical Induction and prove by induction that for $n \geq 1$ one has that $n < 2^n$.