June 29, 2011

Name:

Practice Sheet for Final   Math 5336

1. Define that the relation R is an equivalence on the set A.

2. Define that π is a partition of the set A. How are equivalence relations on A and partitions of A related?

3. What is the partition of the largest equivalence $A \times A$ on A? And what is the partition for the smallest equivalence Δ on A?

4. Let $A = \{a, b, c, d, e, f, g\}$. What are the classes of the smallest equivalence relation that contains the following pairs $\{(a, c), (e, c), (d, f), (g, d), (b, e)\}$?

5. (a) Let $f : A \to B$ be any function from the set A to the set B. Define a relation R_f on A by $(a, b) \in R_f$ if and only if $f(a) = f(b)$. Explain why R_f is an equivalence relation.

   (b) Let $f : \mathbb{R} \to \mathbb{R}$ be the parabola, that is $f(x) = x^2$. What do the equivalence classes look like?

6. (a) Let $A = \{a, b, c, d, e, f, g\}$ and $B = \{1, 2, \}$ and let f be the function for which one has that $f(a) = 1, f(b) = 1, f(c) = 1, f(d) = 2, f(e) = 1, f(f) = 2, f(g) = 2$. What is the partition of the equivalence relation R_f for f?

   (b) Let $f : A \to B$ be a surjection from A to B. Assume that A has $n-$many elements and B has $m-$many elements. What can you say about the number k of equivalence classes for the equivalence relation R_f?

      i. $k = n$;
      ii. $k = m$;
      iii. none of the above.

7. (a) Define that P is a partial order of the set A.

   (b) Show that the relation *a divides b* is a partial order on the set $\mathbb{N}$ of natural numbers. Is there a minimum or maximum of this partial order? Explain your answer.

   (c) Let $(P, \prec)$ be a finite partially ordered set. Explain how the partial order $\prec$ can be extended to a compatible total order $\leq$. Illustrate this process where $P = \{a, b, c, d, e, f\}$ and where $a \prec c, b \prec c, c \prec d, d \prec e, d \prec f, b \prec g$.

8. (a) What does it mean that sets a and b are equivalent? State the Cantor-Bernstein theorem.

   (b) Let $\mathbb{R}^+ = \mathbb{R} \cup \{+\infty\}$ be the set of real numbers extended by a new element, called $+\infty$. Is there a bijection from $\mathbb{R}$ onto $\mathbb{R}^+$? Explain!