Math 6370-01 (14953) (Fall 2015)
Numerical Analysis

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Texts
"Numerical Mathematics" by Alfio Quarteroni, Riccardo Sacco & Fausto Saleri, Springer, Second Edition (ISBN-10: 3642071015, ISBN-13: 978-3642071010)
The Matlab scripts for the examples in the second edition of the book can be downloaded as a tarred gzip file: http://mox.polimi.it/it/progetti/pubblicazioni/qss/programs_QSS_en.tar.gz
Additional text: "Iterative Methods for Linear Systems: Theory and Applications" by M.A. Olshanskii and E.E. Tyrtyshnikov

Objectives
This course is part of a two-course series meant to introduce Ph.D. students in mathematics to the fundamentals of numerical mathematics. The course provides the foundation of numerical methods, the analyze of their basic properties (stability, accuracy, and computational complexity), and demonstrates their performances on examples.

Prerequisites
Graduate standing or consent of instructor. Students should have had a course in Linear Algebra and an introductory course in analysis. Familiarity with Matlab or experience with compiled languages (e.g., Fortran, C, C++) is also required.

Course Policies and Procedures

Grades: Homework (40 percent), Exams (60 percent)

Exams: All three Exams will be given in class (one hour and half) and will be open-book, open-note. Students with a valid excuse for missing up to one exam must provide written documentation to that effect, e.g., a medical certificate.

Homework: There will be regular assignments. The assignments will be both computational and theoretical in approximately 60%-40% ratio. A theoretical home assignment is due in one week after it is assigned in the class (unless a different due date explicitly set in the class). A computational home assignment is due in two weeks (unless a different due date explicitly set in the class).

Course Outline

This is only a tentative agenda for the lectures. Adjustments can be made as we go along. Recommended preparation (before the first class!): Read chapter I of the textbook on your own and make sure you can follow it. Those who don't feel confident in there programming skills I recommend to play with MATLAB and consult, for example, the text Scientific Computing with MATLAB and Octave by A. M. Quarteroni & F. Saleri, Springer (any edition).

Direct methods for solving linear algebraic systems.

Lecture 1: Basic principles of numerical mathematics (well-posedness and condition number of a problem, stability of numerical method, complexity).

Lecture 2: Refreshing some foundations of matrix analysis.

Lecture 3: Stability analysis of linear systems: Condition number of a matrix, a priori and a posteriori analysis.

Lecture 4: The Gauss elimination method (GEM), GEM as a matrix factorization, the effect of rounding errors.

Lecture 5: Banded systems, Pivoting in GEM.

Lecture 6: The Cholesky and QR matrix factorizations.

Lecture 7: Block systems: Block-LU factorization, Inverse of a block-partitioned matrix.

Lecture 8: Sparse matrices, the Cuthill-McKee algorithm.

Exam I

Iterative methods for solving linear algebraic systems.

Lecture 9: Linear iterative methods, convergence, iteration matrix.

Lecture 10: Jacobi, Gauss-Seidel and relaxation methods, convergence results.

Lecture 11: Convergence analysis of the Richardson method. Preconditioning.

Lecture 12: Gradient methods

Lecture 13: The Conjugate gradient method

Lecture 14: Krylov subspace iterative methods

Lecture 15: Ill-posed problems: Least squares and Tikhonov regularization

Finding eigenvalues and eigenvectors.

Lecture 16: Geometrical location of the Eigenvalues, stability and conditioning analysis.

Lecture 17: The power method.

Lecture 18: The QR iterations

Lecture 19: The Jacobi method and the Lanczos method.

Exam II.

Rootfinding and solving systems of non-linear equations.

Lecture 20: Geometrical approaches to rootfindings (bisection, Chord, Secant and Regula Falsi methods).

Lecture 21: General fix-point iterations for non-linear equations.

Lecture 22: Postprocessing techniques and multiple roots case.

Lecture 23: Solution of systems of non-linear equations.

Lecture 24: Methods for unconstrained optimization: direct search, descent, line search methods.

Lecture 25: Methods for unconstrained optimization: Newton-type methods.

Lecture 26: Methods for constrained optimization.

Exam III.