The problem

$$-\Delta\, u = 1 \;\text{ in }\; \Omega = (-1,1)^2, \qquad u = 0 \;\text{ on }\; \partial\Omega$$

is discretized with linear finite elements or second order finite differences (5-point stencil) on a uniform grid, with mesh size $h = 2/N$. The matrix of the resulting system of linear algebraic equations can be found on page 35 of the lecture notes.
For $N = 16, 32, 64, 128$, solve the system using
1) The Gauss elimination method;
2) The Richardson method, with $\tau_{opt} = \frac{2}{\lambda_{\min}+\lambda_{\max}}$, $\lambda_{\min} = 8h^{-2}\sin^2(\pi\frac{h}{4})$, $\lambda_{\min} \approx 8h^{-2}$;
3) The Gauss-Seidel method;
4) The Conjugate Gradient method;
Report CPU time for 1)-4) and final iteration numbers for 2)-4). The stopping criterion for 2)-4) is the relative reduction of the ℓ^2 norm of residual by 10^9. Initial guess is zero. Draw conclusions about performance and complexity of the methods.

Verify the accuracy of computed discrete solutions by comparing them at x=(0,0) to the known analytical solution

$$u(x,y) = \frac{(1-x^2)}{2} - \frac{16}{\pi^3}\sum_{k=0}^{\infty}\left\{\frac{\sin((2k+1)\pi(1+x)/2)}{(2k+1)^3\mathrm{sh}((2k+1)\pi)}\times\right.$$
$$\left.\Big(\,\mathrm{sh}((2k+1)\pi(1+y)/2) + \mathrm{sh}((2k+1)\pi(1-y)/2)\,\Big)\right\}$$