

A SHORT PAPER

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1. INTRODUCTION

We will exemplify here how to number, label and refer to sections, theorems etc. in a \LaTeX paper. Along the way, we will prove a remarkable result — see Section 3.

2. PRELIMINARIES

We want to prove that the even numbers form a ring with the operations of addition and multiplication. We begin with two propositions.

Proposition 2.1. *The sum and difference of two even numbers is even.*

Proof. Let

$$(2.1) \quad n = 2k$$

and

$$(2.2) \quad n = 2\ell$$

be two even numbers. Hence $k, \ell \in \mathbb{Z}$.

Then

$$n + m = 2k + 2\ell = 2(k + \ell)$$

$$n - m = 2k - 2\ell = 2(k - \ell)$$

therefore $n \pm m$ is a multiple of two as well. □

Proposition 2.2. *The product of two even numbers is even.*

Proof. Let n and m be as in (2.1), (2.2), with $k, \ell \in \mathbb{Z}$. Then

$$n \cdot m = (2k)(2\ell) = 2(2k\ell),$$

hence $n \cdot m$ is even (actually, even a multiple of 4). □

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3. THE MAIN THEOREM

Theorem 3.1 (Our Main Result). *The set $\mathcal{E} := \{2k \mid k \in \mathbb{Z}\}$ forms a ring.*

Proof. Since \mathbb{Z} is a ring and $\mathcal{E} \subset \mathbb{Z}$, we only have to check that

$$(3.1) \quad x, y \in \mathcal{E} \Rightarrow x - y \in \mathcal{E},$$

$$(3.2) \quad x, y \in \mathcal{E} \Rightarrow xy \in \mathcal{E}.$$

But (3.1) was proven in Proposition 2.1, and (3.2) in Proposition 2.2. \square

4. REMARKS

Following a similar approach, one can show that the following rules apply for addition (see Table 4.1) and multiplication:

+	even	odd
even	even	odd
odd	odd	even

TABLE 4.1. mod 2 addition table

·	even	odd
even	even	even
odd	even	odd

5. BIBLIOGRAPHY, BIBTEX

You can also add references to your paper, e.g. Strassen [1] and Takahashi [Ta72] (of course, do not mix numerical and alphabetical labels).

One very convenient way to do this is BibTeX. One creates a database of bibliographical references, and then formats it easily in the desired style. See more under the BibTeX entry.

REFERENCES

- [1] V. Strassen. Almost sure behavior of sums of independent random variables and martingales. *Proc. 5th Berkeley Symp. Math. Statist. Probab.*, **2** (1967), 315–343.
- [Ta72] S. Takahashi. Notes on the law of the iterated logarithm. *Studia Sci. Math. Hung.* **7** (1972), 21–24.