

HWK 4

$$\textcircled{1} \left. \begin{aligned} \frac{d^2\phi}{dx^2} + \lambda\phi &= 0 \\ \phi(0) &= 0 \\ \phi(L) &= 0 \end{aligned} \right\} \Rightarrow p=1, q=0, G=1$$

$$\lambda = \frac{-p(L)\phi'(L) + p(0)\phi'(0) + \int_0^L \left(\frac{d\phi}{dx}\right)^2 dx}{\int_0^L \phi^2 dx} = \frac{\int_0^L \left(\frac{d\phi}{dx}\right)^2 dx}{\int_0^L \phi^2 dx} \geq 0$$

↑
Rayleigh quotient

Now, $\int_0^L \left(\frac{d\phi}{dx}\right)^2 dx = 0$ only if $\phi = \text{const.}$

But if $\phi = \text{const.}$, from $\phi(0) = 0$ & $\phi(L) = 0$, we see that $\phi \equiv 0$. However $\phi \equiv 0$ is not an eigenfunction.

So, we conclude that $\lambda > 0$.

$\textcircled{2}$ See Problem 5 solution on Quiz 6 Sample Answer Key.