Homework 2: January 19, 2017

1. Read Section 1.3: "Boundary Conditions" (pages 11-13 in 5th edition).

2. Read "Diffusion of a chemical pollutant" in Section 1.2. (Not covered in class.)

3. Problem 1.2.8: If $u(x,t)$ is known, give an expression for the total thermal energy contained in a rod of length L, and cross-section A.

4. If the distribution of temperature in a rod of length 2, radius 1, density $\rho = 3$ and specific heat $c = 4$ is given by $u(x,t) = 3xt$, what is the total thermal energy of the rod at time t?
(Assume that all the units are in correspondence so that no conversion is needed.)

5. Write the initial-boundary value problem for the heat equation describing propagation of heat in a 1D rod of length 2, with thermal diffusivity 4, initial temperature $u_0(x) = 3x, x \in (0,2)$, the left end insulated, and the right end of the rod having a prescribed oscillating temperature $100 - \cos t$.
(Assume that all the units are in correspondence so that no conversion is needed.)

6. Problem 1.3.2, pg 13:
Two 1D rods of different materials joined at $x = x_0$ are said to be in perfect thermal contact if the temperature is continuous at $x = x_0$:

$$u(x_0-,t) = u(x_0+,t)$$

and no heat energy is lost at $x = x_0$ (i.e., the heat energy flowing out of one flows into the other). What mathematical equation represents the latter condition at $x = x_0$? Under what special condition is $\partial u/\partial x$ continuous at $x = x_0$?

7. What is Fourier's Law?

8. Write the boundary conditions describing a 1D rod with thermal conductivity $k_0 = 2$, whose left end has a prescribed heat flux given by $4 \sin t$, and the right end is exposed to air at temperature $4t$ (increasing in time), with the heat transfer coefficient $H = 0.1$.