Homework 6: Feb 9, 2017

1. (Problem 2.3.3.) Consider the heat equation $u_t = ku_{xx}$, subject to the boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$. Solve the initial boundary value problem if the temperature is initially:

(a) $u(x,0) = 6\sin\frac{9\pi x}{L}$

(b) $u(x,0) = 3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}$

(c) $u(x,0) = 2\cos\frac{3\pi x}{L}$

2. (Problem 2.3.4) Consider $u_t = ku_{xx}$, subject to $u(0,t) = 0$, $u(L,t) = 0$ and $u(x,0) = f(x)$.

(a) What is the total heat energy in the rod as a function of time?

(b) What is the flow of heat energy out of the rod at $x = 0$? at $x = L$?

3. (Problem 2.3.6) Evaluate

$$\int_0^L \cos\frac{n\pi x}{L}\cos\frac{m\pi x}{L}dx \quad \text{for } n \geq 0, m \geq 0.$$

Use trigonometric identity

$$\cos a\cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right].$$

Be careful if $a - b = 0$ or $a + b = 0$.

4. Find the Fourier sine series for the function $f(x) = 1$.

5. Using the answer from problem 4, solve the following initial-boundary value problem:

$$\begin{aligned} u_t &= u_{xx}, \quad x \in (0,L), t > 0 \\ u(0,t) &= 0, \quad t > 0, \\ u(L,t) &= 0, \quad t > 0, \\ u(x,0) &= 1, \quad x \in (0,L). \end{aligned}$$

NOTE: PROBLEMS 4 and 5 WILL NOT BE COVERED IN QUIZ 3.