NAME (please print): ______________________

Quiz 5: Due March 21 (beginning of class)

1. (30 pts) Solve the following initial-boundary value problem:

$$\begin{aligned} u_{tt} &= 4u_{xx},\ x \in (0,1), t > 0, \\ u(0,t) &= 0, \\ u(1,t) &= 0, \\ u(x,0) &= 0 \\ u_t(x,0) &= 3\sin 2\pi x. \end{aligned}$$

2. (40 pts) Solve the following initial boundary value problem for the damped string:

$$\begin{aligned} u_{tt} &= u_{xx} - u_t,\ x \in (0,1), t > 0, \\ u(0,t) &= 0, \\ u(1,t) &= 0, \\ u(x,0) &= \sin \pi x \\ u_t(x,0) &= 0. \end{aligned}$$

Warning: Be very careful when dealing with the initial data $u_t(x,0) = 0$ for this problem with damping!

3. (30 pts) By using the following trigonometric formulas

$$\sin\alpha \sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right], \quad \sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta) - \sin(\alpha+\beta)\right],$$

and the Fourier series expression for the general solution of the vibrations of a linearly elastic string fixed at the end points $x = 0$ and $x = L$:

$$u(x,t) = \sum_{n=1}^{\infty} \left(A_n \cos\frac{n\pi ct}{L} + B_n \sin\frac{n\pi ct}{L} \right) \sin\frac{n\pi x}{L}$$

show that the general solution $u(x,t)$ can be written as a sum of two functions $F(x-ct)$ and $G(x+ct)$ where $F(x-ct)$ describes a forward traveling wave moving with speed c, and $G(x+ct)$ describes a backward traveling wave moving with speed c.