UH - Math 3330 - Dr. Heier - Sample Midterm Exam - Fall 2009
Time: 70 min

1. (a) (10 points) Prove that $(A \cup B) \backslash C = (A \backslash C) \cup (B \backslash C)$.
(b) (10 points) What is $(A \backslash B) \cap B$? Prove your answer.

2. Let $f : \mathbb{N} \to \mathbb{N}, n \mapsto n + 1$.

(a) (5 points) Is f injective?

(b) (5 points) Is f surjective?

(c) (5 points) Give a left-inverse to f (if it exists).

(d) (5 points) Give a right-inverse to f (if it exists).

3. (20 points) Let $x, y \in \mathbb{Z}$. Let $x \sim y$ if and only if $4|x + 3y$. Prove that $\sim$ is an equivalence relation.

4. (a) (10 points) Prove: $\forall$ integers $n \geq 3 : 1 + 2n < 2^n$.
(b) (10 points) Prove: $\forall$ integers $n \geq 1 : 6|n^3 + 5n$.

5. (a) (10 points) Find the gcd of $a = 124$ and $b = 52$ via factorization into primes.
(b) (10 points) Find the same gcd again by using the Euclidean algorithm as discussed in class. Also, find m, n such that $gcd(a, b) = ma + nb$.