1. (a) (10 points) Find all solutions of

$$5x + 43 \equiv 15 \ (\text{mod } 22).$$

(b) (10 points) Find all solutions of the system

$$x \equiv 1 \ (\text{mod } 5)$$
$$x \equiv 2 \ (\text{mod } 9).$$

2. (a) (10 points) Find the solution of the following equation in $\mathbb{Z}_{20}$:

$$[9][x] = [14].$$

(b) (10 points) Find the solution of the following system of equations in $\mathbb{Z}_7$:

$$[2][x] + [3][y] = [1]$$
$$[3][x] + [2][y] = [3].$$

3. (a) (10 points) Let $(G, *)$ be a group. Give a definition for a non-empty subset H of G to be a subgroup.

(b) (10 points) Is the set $\{-1, 1\}$ a subgroup of $(\mathbb{R}\backslash\{0\}, \cdot)$? Give complete details in your answer.

4. (a) (10 points) Let a, b be elements of a finite group G. Prove that a and $b^{-1}ab$ have the same order.

(b) (10 points) Let a, b be elements of a group G. Prove that G is abelian if and only if $abab = aabb$.

5. (20 points) Prove that $(\mathbb{Z}, *)$ is an abelian group with the group operation

$$x * y = x + y - 1.$$