UH - Math 4377/6308 - Dr. Heier - Fall 2011
HW 1
Due 08/31, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 6\}$. Explicitly write down the sets

$$A \cup B, \; A \cap (B \cup C), \; B \cap (A \backslash B), \; A \times C.$$

2. (3 points) Let $x, y \in \mathbb{Z}$. Prove or disprove that the following relations are equivalence relations.

(a) $x \sim y$ if and only if $x - y$ is less than 10.

(b) $x \sim y$ if and only if $x \cdot y \geq 0$.

(c) $x \sim y$ if and only if $x - y$ is even.

3. (3 points) Let $f : \{0, 1, 2, 3, 4\} \to \mathbb{N}, n \mapsto n^3 - n$.

(a) Find the domain, codomain and range of f.

(b) Is f one-to-one?

(c) Is f onto?

4. (1 point) Give an example of a real interval I on which the standard sin function is one-to-one with the additional property that sin is not one-to-one on any set strictly containing I. Explain your answer carefully, assuming standard facts about sin without proof.

5. (1 point) Let a be an arbitrary element in a field. Prove that the additive inverse $-a$ is unique. (Hint: You may use without proof the Cancellation Laws Theorem.)

6. (0.5 points) Let $z = 1 + 3i$, $w = 1 - 2i$. Write $\bar{z}$, $z + w$, zw, $|z|$, $\frac{1}{z}$ in the form $a + bi$.

7. (0.5 points) Solve $z^2 - 4z + 13 = 0$ in $\mathbb{C}$.

8. (1 extra credit point) Let $x, y \in \mathbb{Z}$. Let $x \sim y$ if and only if $y + 4x$ is an integer multiple of 5. Prove that $\sim$ is an equivalence relation.