UH - Math 4377/6308 - Dr. Heier - Fall 2012
HW 3
Due 09/19, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. Determine if the following subsets of $\mathbb{R}^3$ are subspaces.

(a) (0.5 points) $\{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 - 3a_3 = 0\}$

(b) (0.5 points) $\{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 - 2a_2 + a_3 = 1\}$

(c) (0.5 points) $\{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 = a_3\}$

(d) (0.5 points) $\{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 = 5a_3 \text{ and } 4a_2 = a_1 + a_3\}$

2. Determine if the following subsets of the vector space of 2×2 matrices with real entries are subspaces. You may assume as true that the set of 2×2 matrices with real entries forms a vector space with the usual addition and scalar multiplication.

(a) (1 point) $\left\{ \begin{pmatrix} a_1 & a_2 \\ a_3 & 0 \end{pmatrix} : a_1, a_2, a_3 \in \mathbb{R} \right\}$

(b) (1 point) $\left\{ \begin{pmatrix} a_1 & a_2 \\ a_2 & a_1^2 \end{pmatrix} : a_1, a_2, a_3 \in \mathbb{R} \right\}$

3. (1 point) A real-valued function f defined on the real line is called an *even function* if $f(t) = f(-t)$ for each real number t. Prove that the set of even functions is a subspace with the usual addition and scalar multiplication for functions. You may assume as true that the set of real-valued functions f defined on the real line is a vector space with the usual addition and scalar multiplication for functions.

4. (1 point) Let W_1, W_2 be two subspaces of a vector space V. Prove that the intersection $W_1 \cap W_2$ is also a subspace of V.

5. (1 point) Section 1.3, Problem 18.

6. (2 points) Let V be the vector space of all functions $f : \mathbb{R} \to \mathbb{R}$. Let $W_1 = \{f : \mathbb{R} \to \mathbb{R} | f(5) = 0\}$. Prove that W_1 is a subspace of V. Find a subspace $W_2 \subset V$ such that $V = W_1 \oplus W_2$. Prove all your statements. (This is the problem I posed in class, and I couldn't resist putting it on the homework.)

7. (1 point) Section 1.3, Problem 28 (Work with $F = \mathbb{R}$ only. This allows you to disregard the half-sentence "Now assume that F is not of characteristic 2 (see Appendix C),".)

8. (1 extra point) Let W_1, W_2 be two subspaces of a vector space V. Prove that the union $W_1 \cup W_2$ is a subspace of V if and only if $W_2 \subseteq W_1$ or $W_1 \subseteq W_2$.